
Tomographic reconstruction: a guided tour

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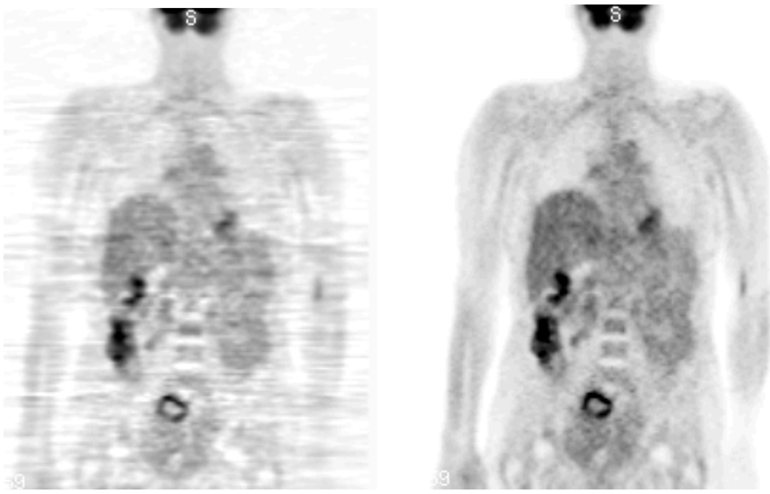
Learning objectives

Understanding tomographic reconstruction

- How PET, SPECT, CT images are obtained from the signal delivered by the scanners
- Understand the differences between analytical and iterative reconstruction
- Knowing key parameters in tomographic reconstruction and how they impact the resulting images



Very different images can be obtained from the same original data. Why ????

Learning objectives

Understand the maths and the practice of tomographic reconstruction

What is RAMLA 3D ? Isn't tomographic reconstruction always 3D ?

What is a sinogram ?

Which number of iterations should be used in iterative methods?

What does rebinning mean?

Your questions …

Outline

- Introduction
  - What is tomography?
  - Transmission tomography
  - Emission tomography
  - Why is tomographic reconstruction so difficult?

- Basic concepts
  - Projection
  - Radon transform
  - Sinogram

- Analytical reconstruction
  - Principle
  - Central slice theorem
  - Filtered backprojection
  - Filters

- Iterative reconstruction
  - Principle
  - Matrix system
  - MLEM, OSEM, RAMLA, aso
  - Regularization

- « Fully 3D » reconstruction
  - Principle
  - Rebinning methods

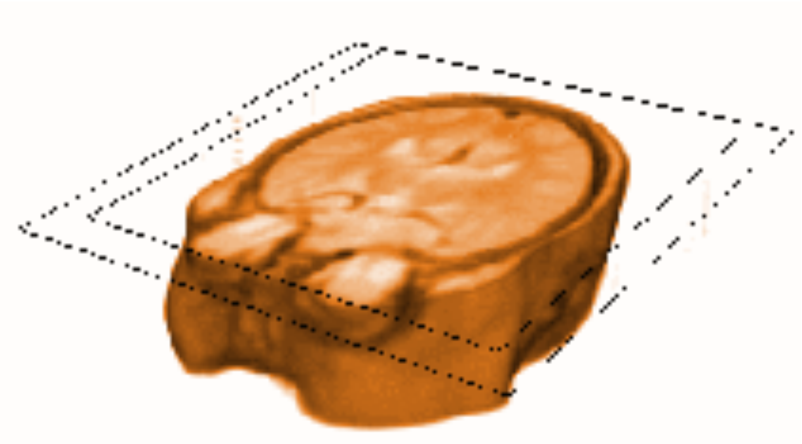
- Questions / Discussion

Please interrupt and ask questions whenever needed

Introduction: what is tomography?

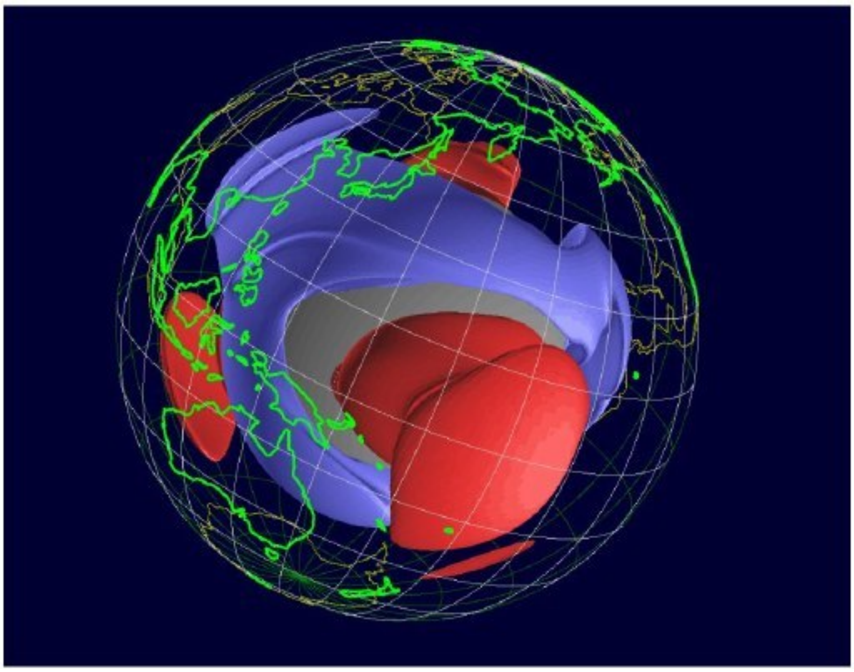
- Tomos : slice (greek)
- Graphia : writing

- Mapping an internal parameter of an “object” using cross sections or slices, based on external non-invasive measurements AND on computer-assisted calculations



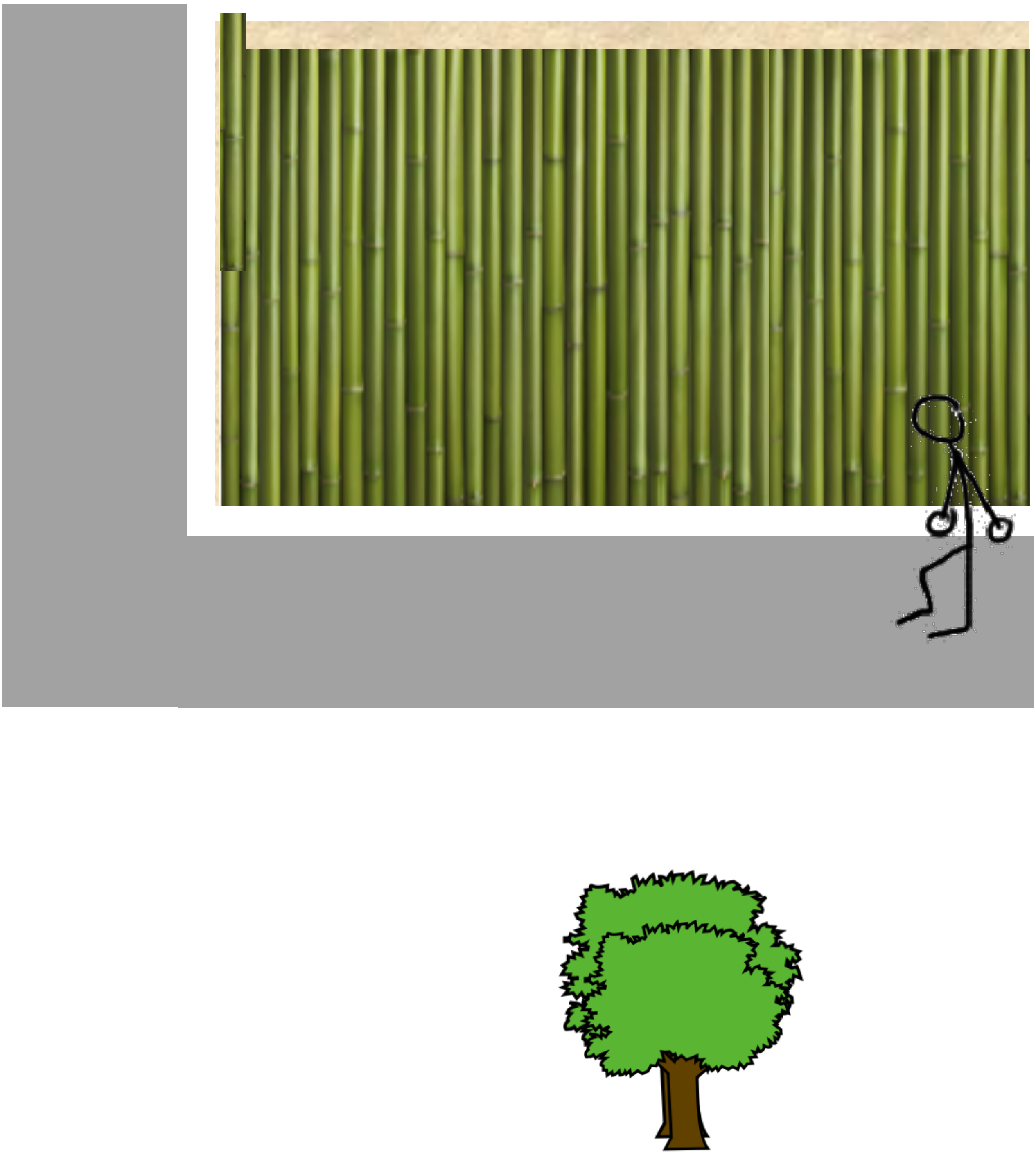
Introduction: what is tomography?

- An approach to probe “objects” that cannot be directly sliced or sampled. Many application fields:
    - non destructive testing
    - geophysics (geological layers, oceans)
    - astrophysics
    - medical imaging



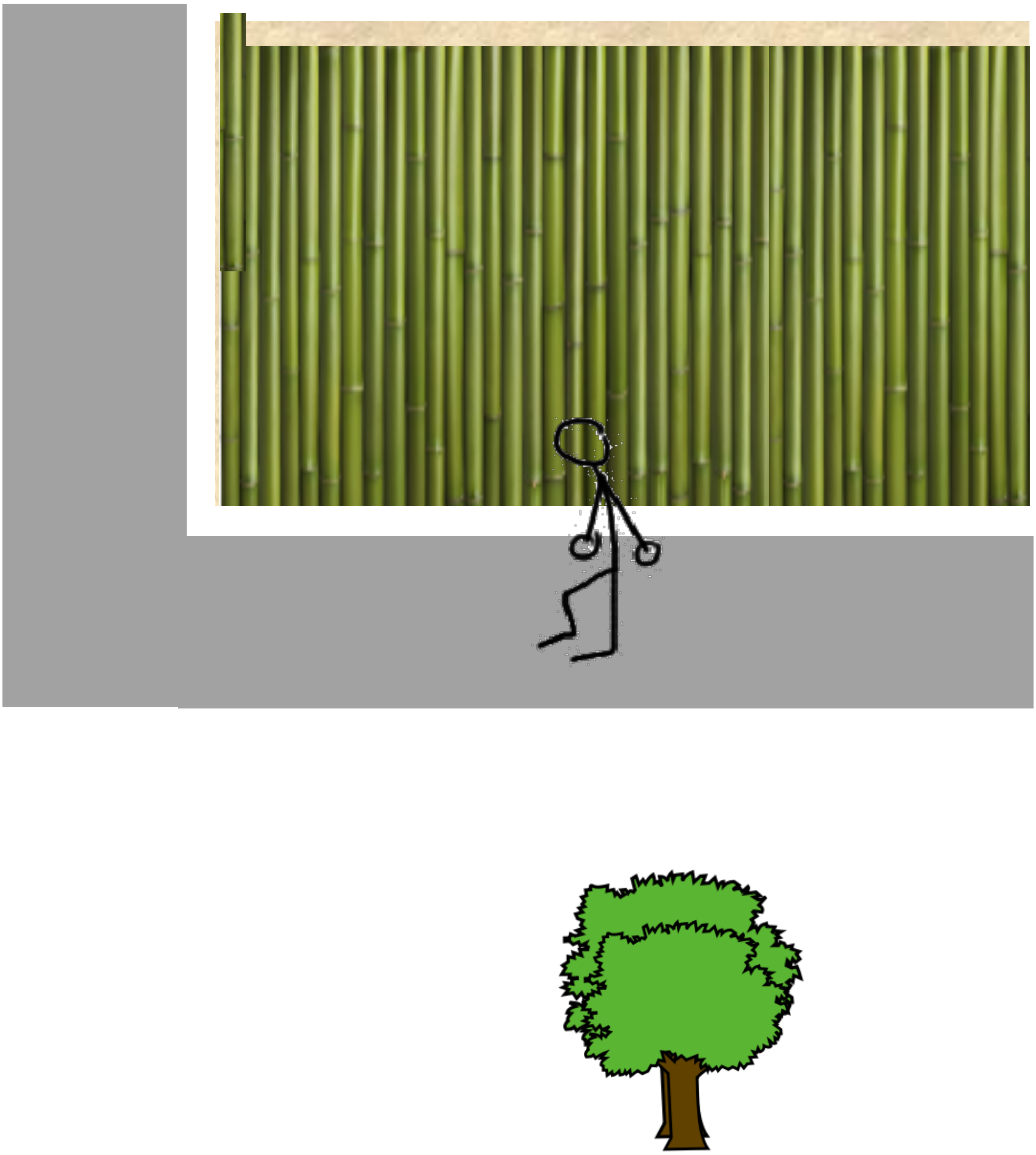
Introduction: everyday tomography (1)

Mapping from partial views



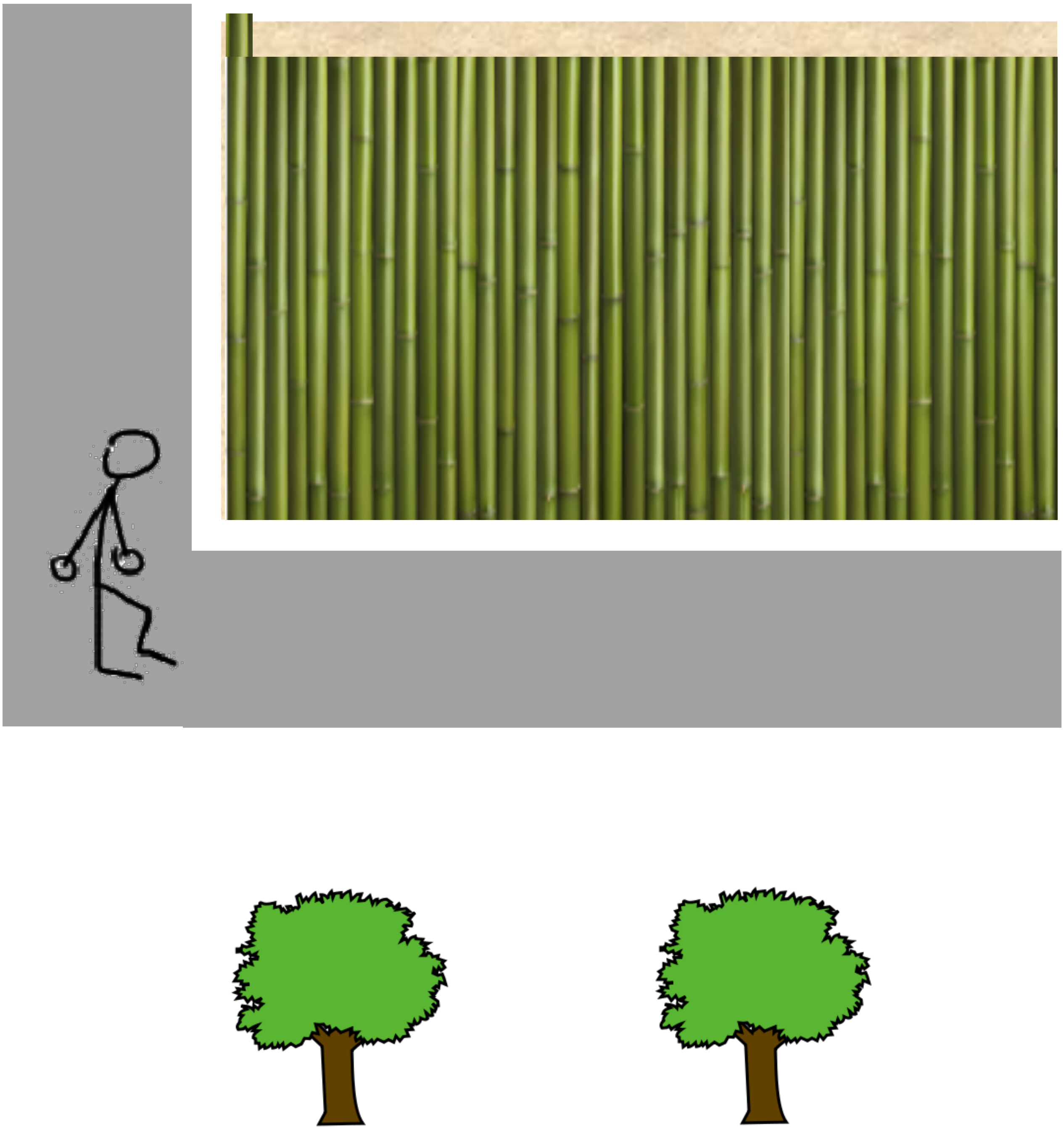
Introduction: everyday tomography (1)

Mapping from partial views

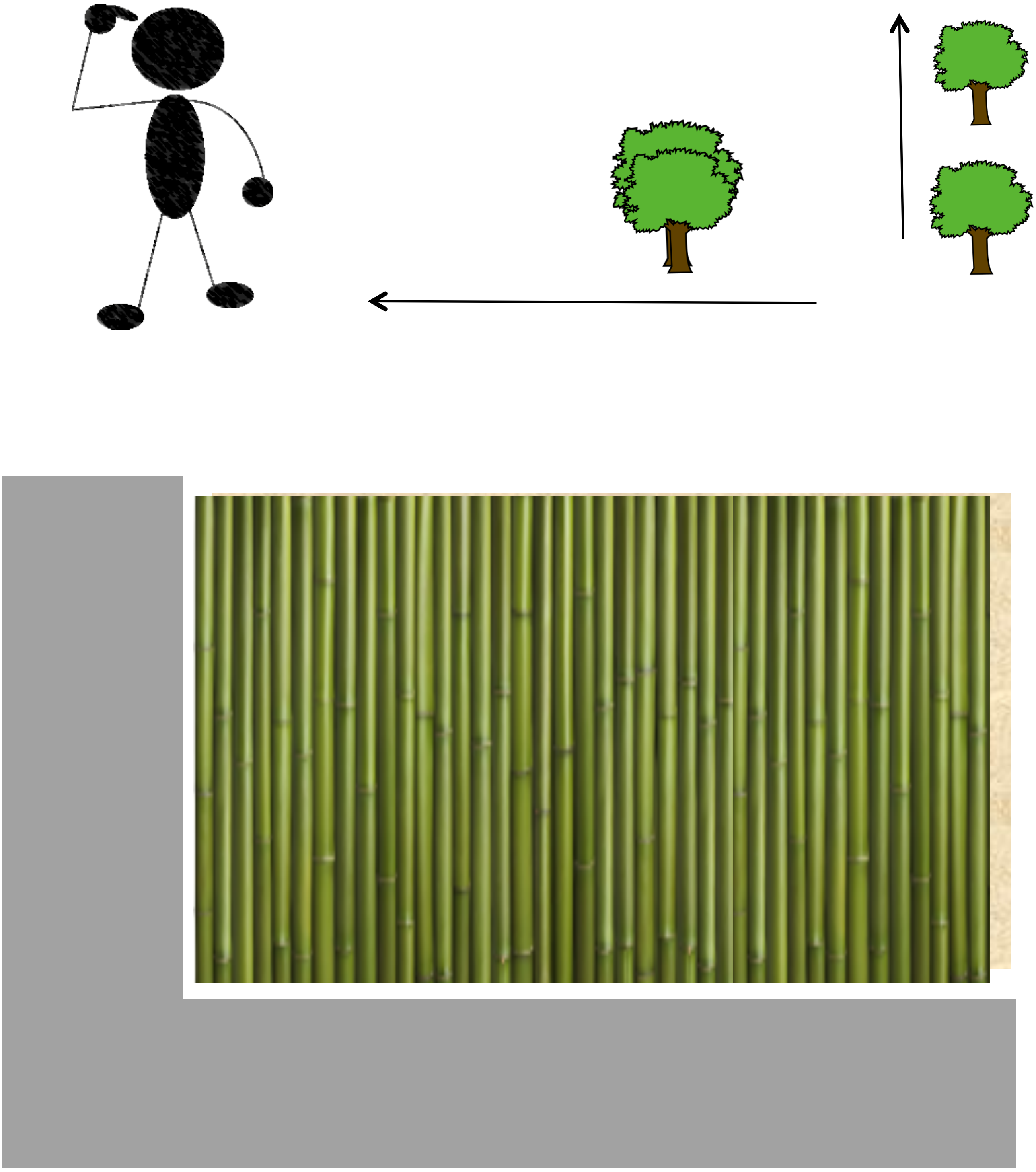


Introduction: everyday tomography (2)

Mapping from partial views

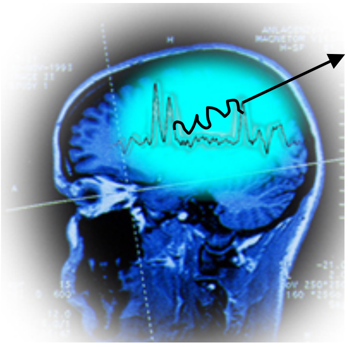


Introduction: everyday tomography (3)

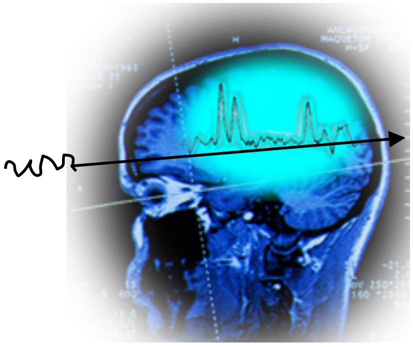


Tomographic reconstruction is a systematic approach to solve that sort of problem

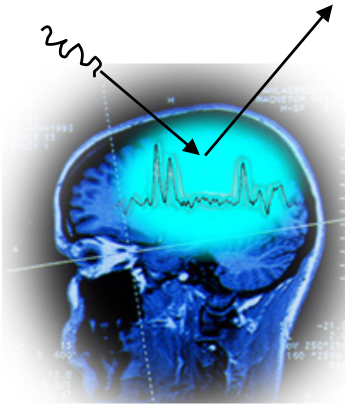
Medical tomography: three types



Emission tomography



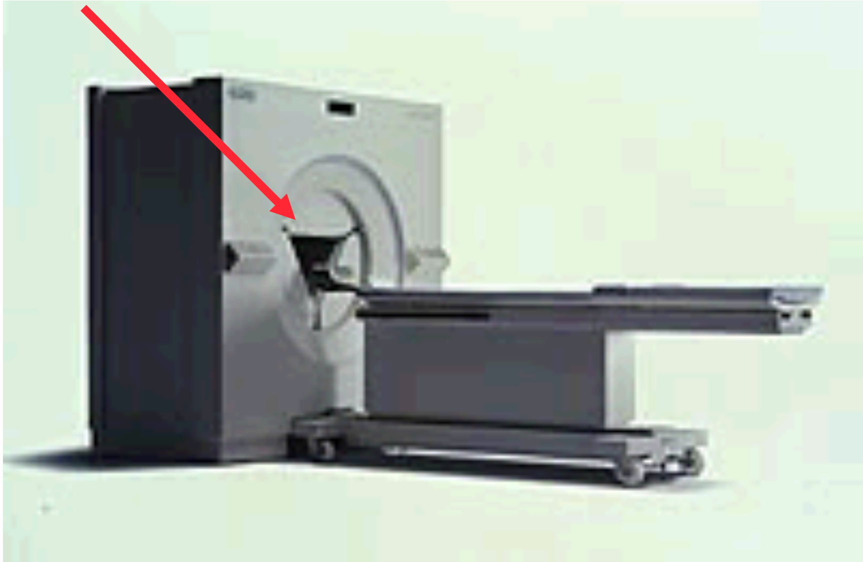
Transmission tomography



Optical tomography
(mostly preclinical)

Medical imaging

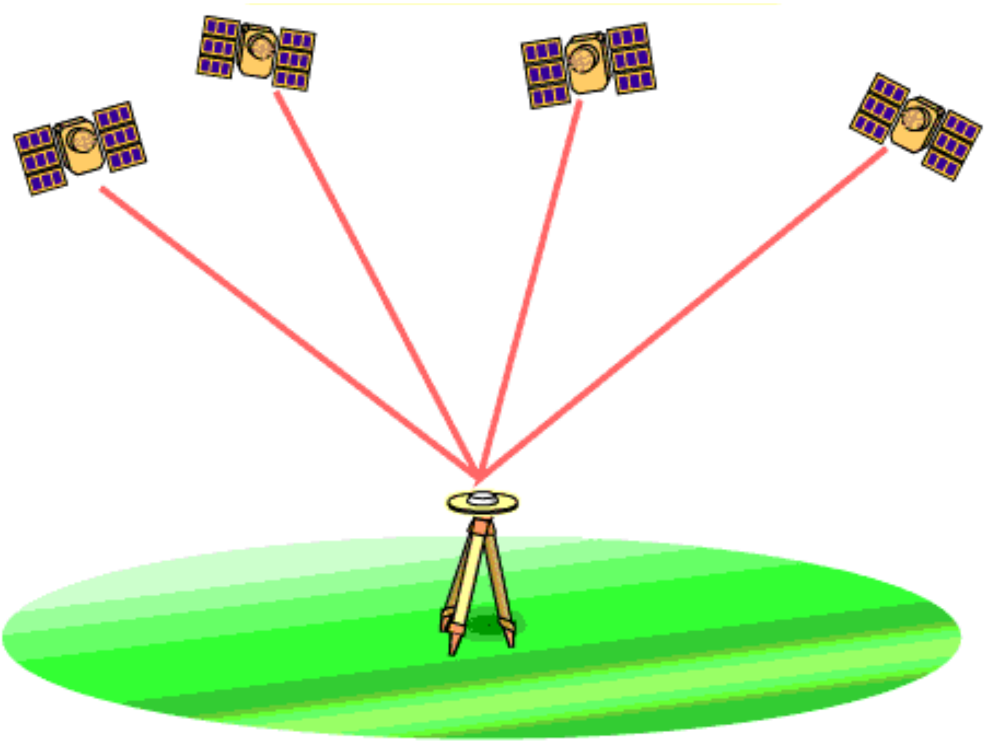
- Measurement of emitted or transmitted radiations using a CT scanner, a gamma camera, a positron emission tomography scanner or a probe (optical tomography)



- Data processing to create images from the measured signal

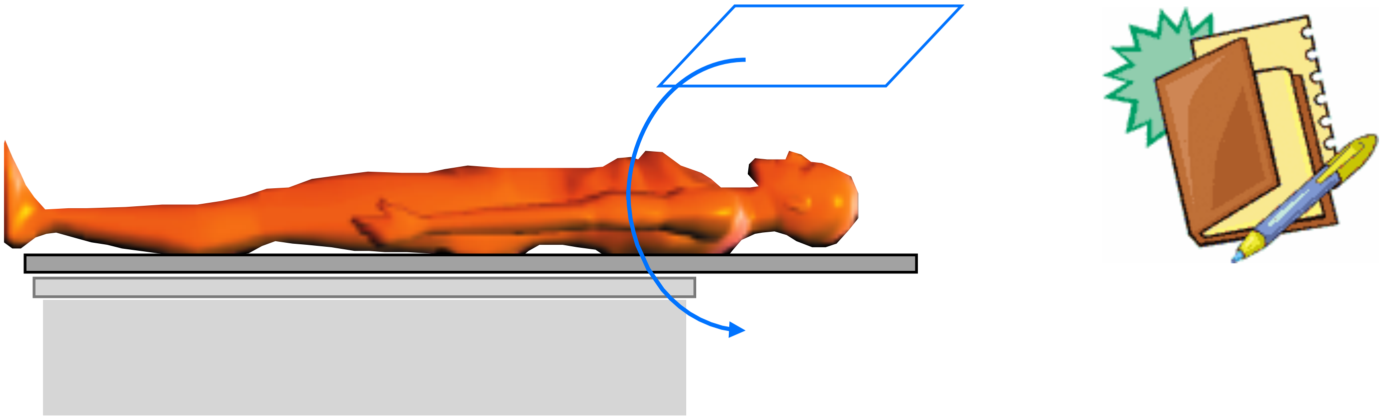
Key point

- Measurements at different angular positions: different views of the same object



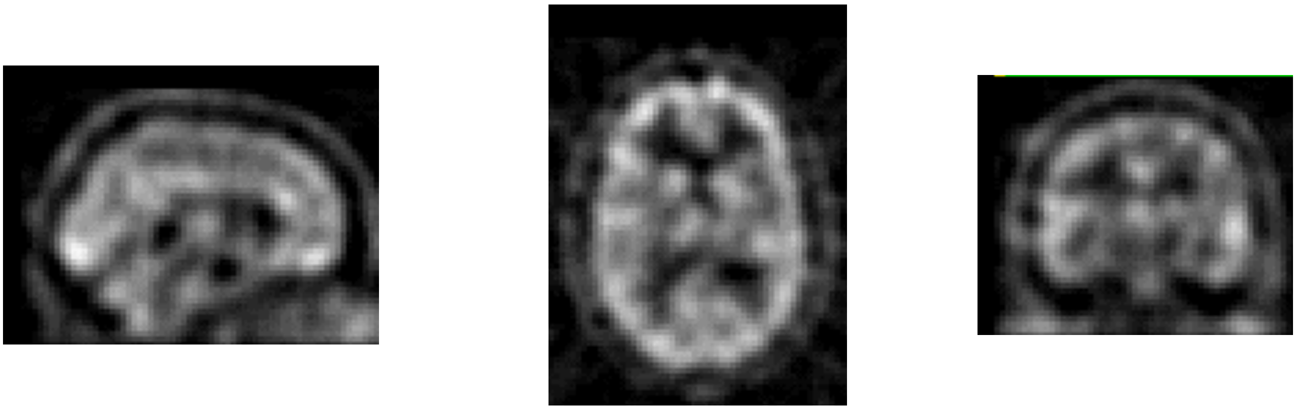
Medical imaging

measurements= projections



Integral measurements at different angles
projections

Data processing

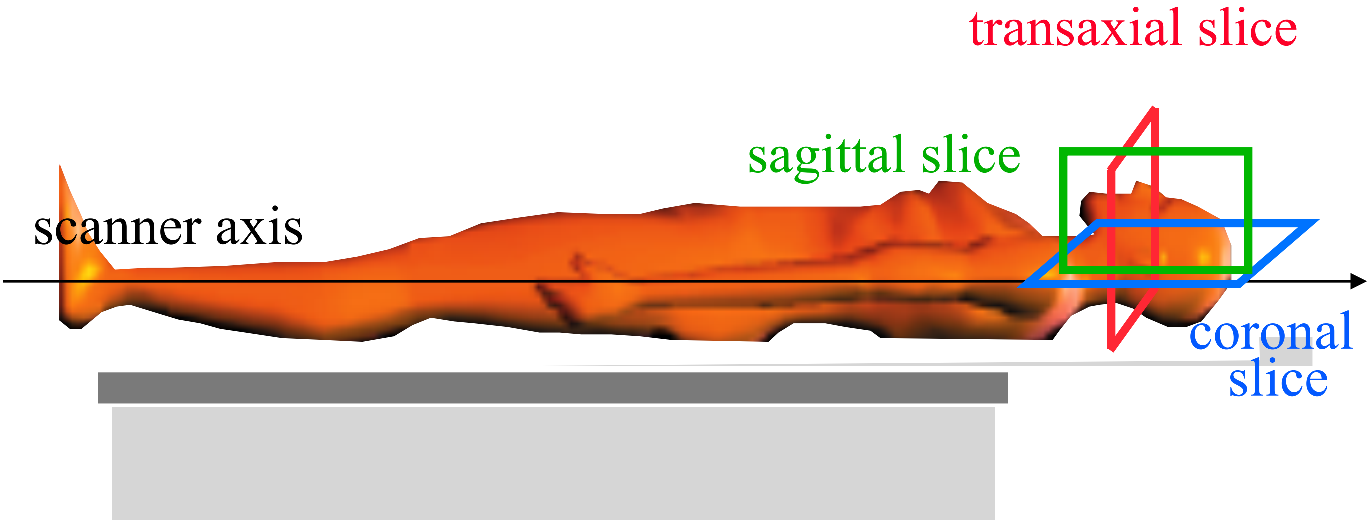


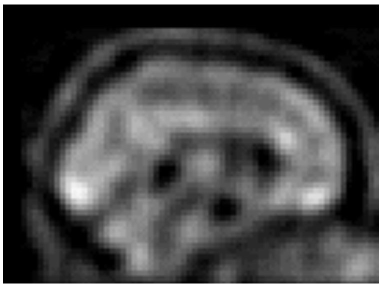
sagittal transaxial coronal

Reconstruction of slices using 3 preferred directions

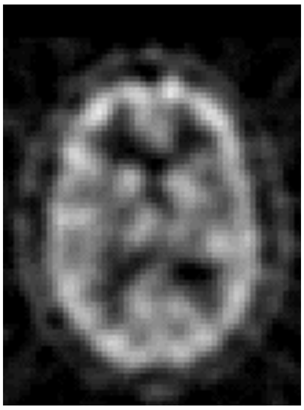
3D imaging: any oblique slice can be obtained

Definition of slice orientation

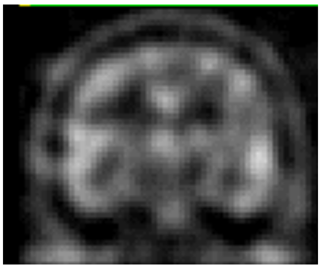




sagittal



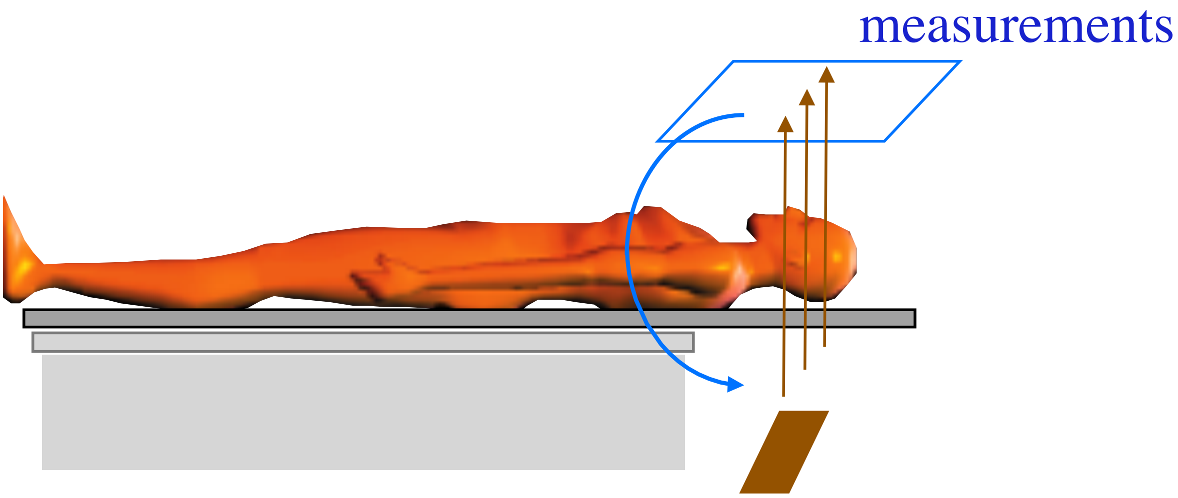
transaxial



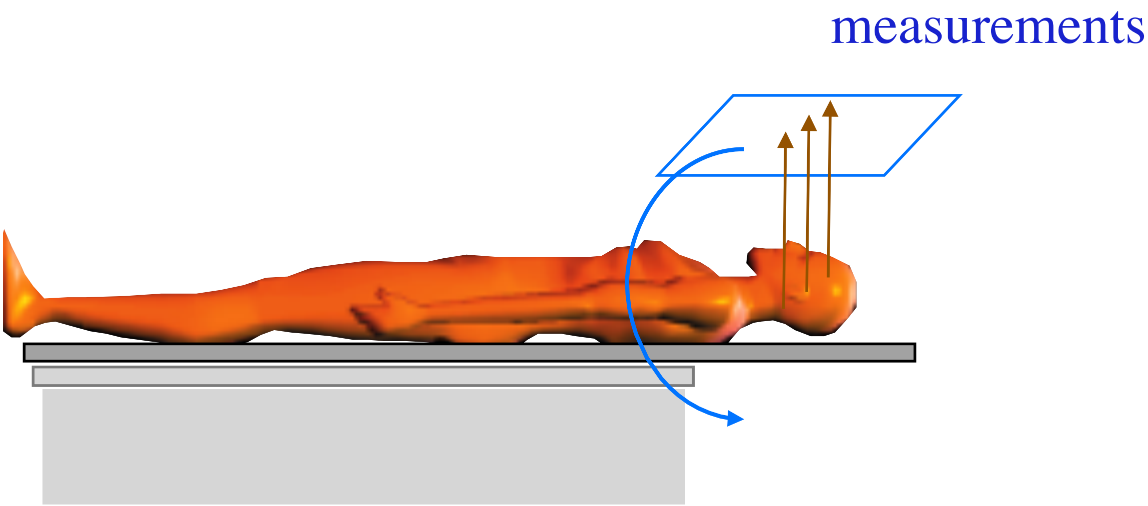
coronal

Two main types of measurement

- Transmission tomography

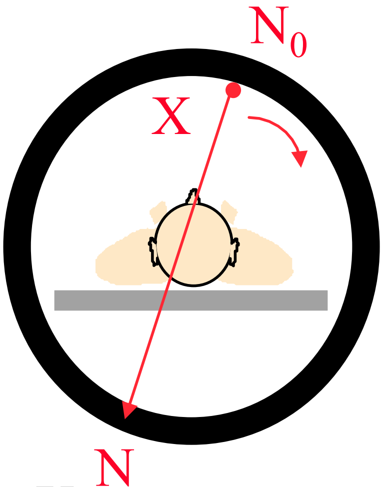


- Emission tomography



Transmission tomography devices

- Source external to the patient



X-ray scanner

Gives information on how X-rays are transmitted by or travel through the tissues, ie on the attenuation properties of the tissues

Transmission tomography: a closer look

- Projection of the transmitted radiations

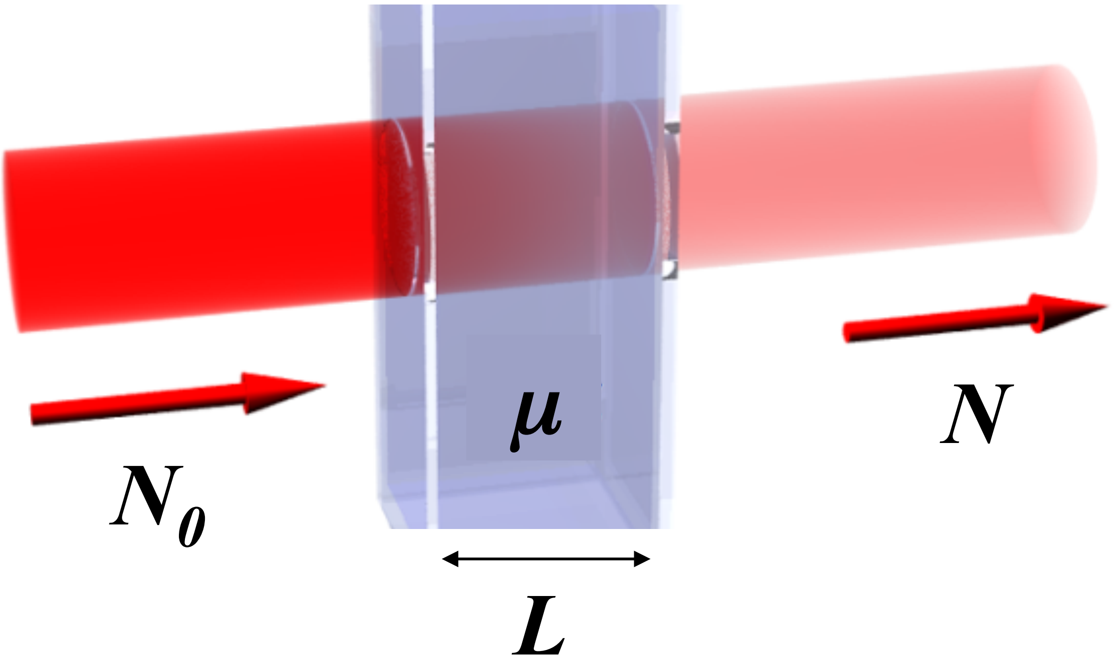
If measured signal intensity ~ source signal intensity :
⇒ almost no attenuation: lungs?

If measured signal intensity <<< source signal intensity :
⇒ lots of interaction between X-rays and matter : tissue with high electron density, eg bone ?

Tomography reconstruction will give you the exact attenuation properties of the tissues

What is attenuation?

- Expressed as an attenuation coefficient, μ, in cm^{-1}



$$N = N_0 \exp(-\mu L)$$

Beer-Lambert law

In water, at 140 keV : $\mu = 0.15\ cm^{-1}$

What percentage of 140 keV photons after 20 cm of water ?

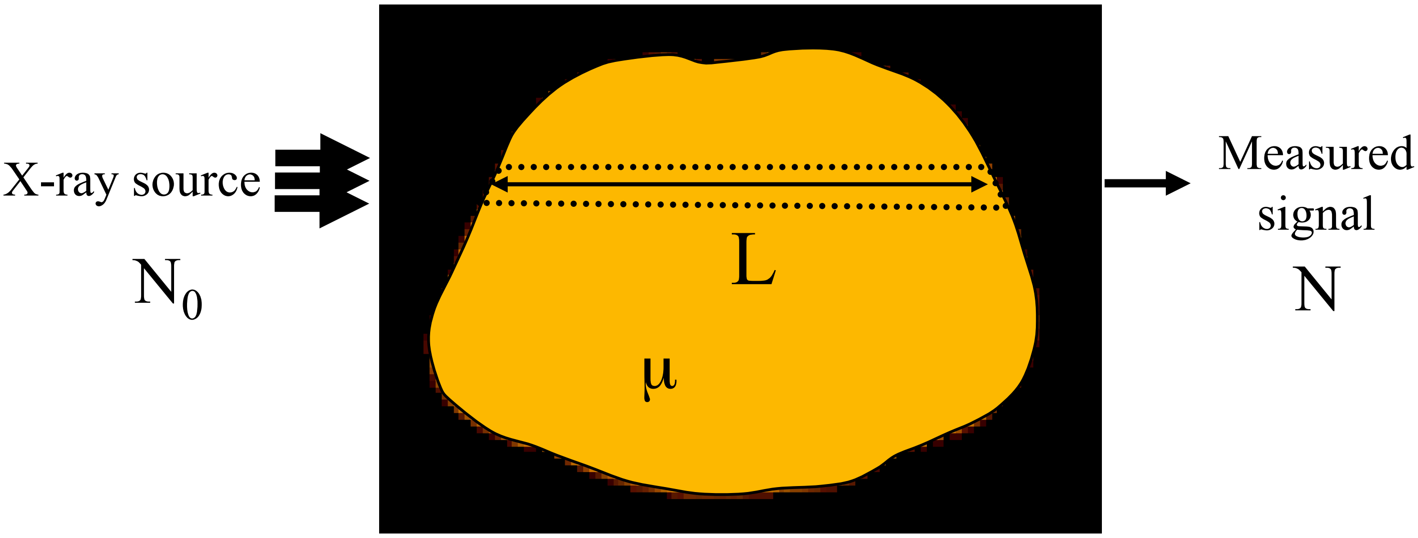
$$N = N_0 \exp(-0.15 \times 20) = 0.05\ N_0\text{, ie } 5\%$$

Which tissue if 45% of 140 keV photons are detected after going through 20 cm of tissue?

$$N/N_0 = 0.45 = \exp(-\mu \times 20)$$
$$20\mu = -\ln 0.45 \Rightarrow \mu = 0.04\ cm^{-1}\text{ (lungs)}$$

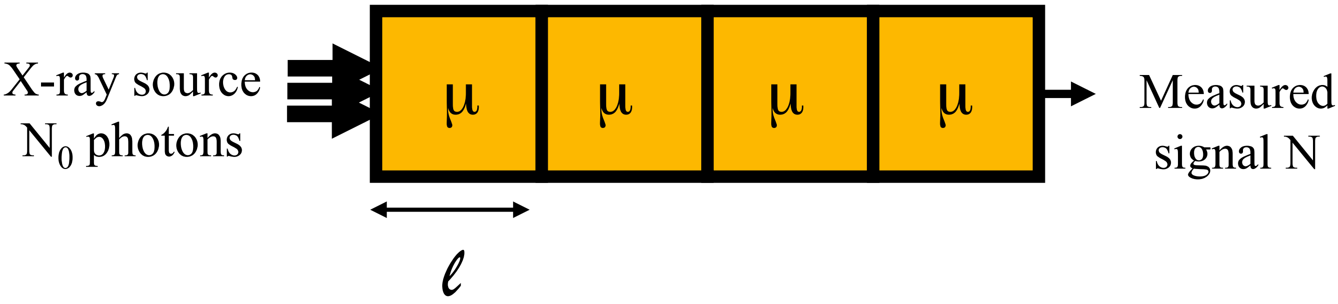
Modeling ET measurements

- Attenuation of an X-ray source in a uniform medium of attenuation coefficient μ (cm^{-1})



$$N = N_0 \exp(-\mu L)$$

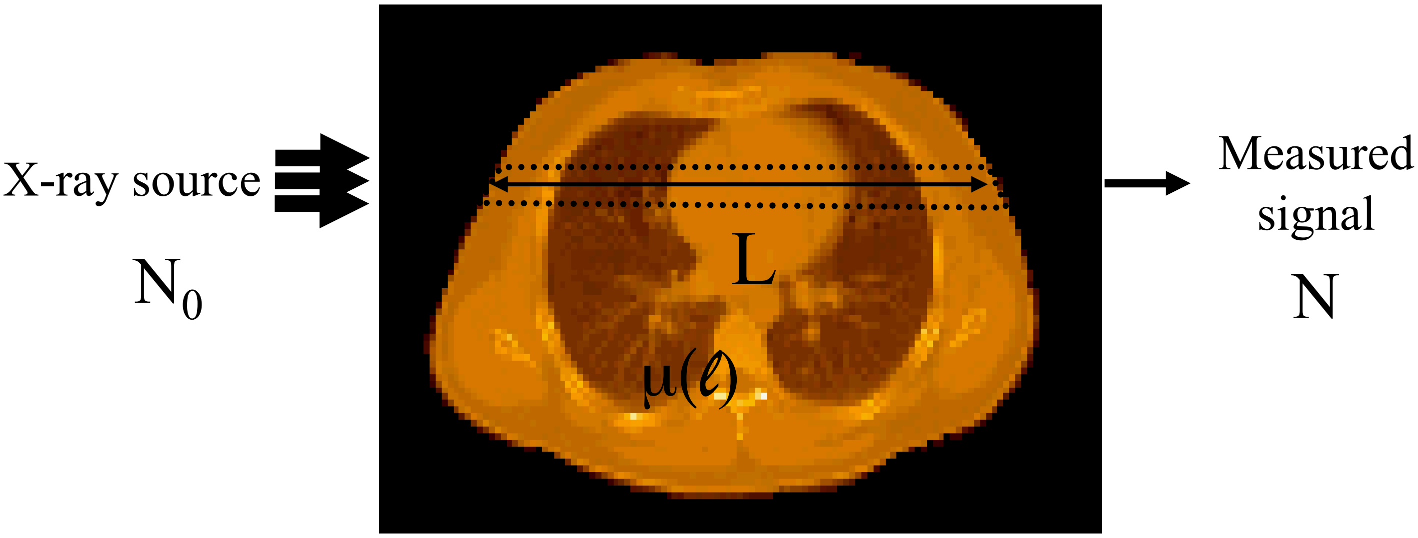
- Discrete expression :



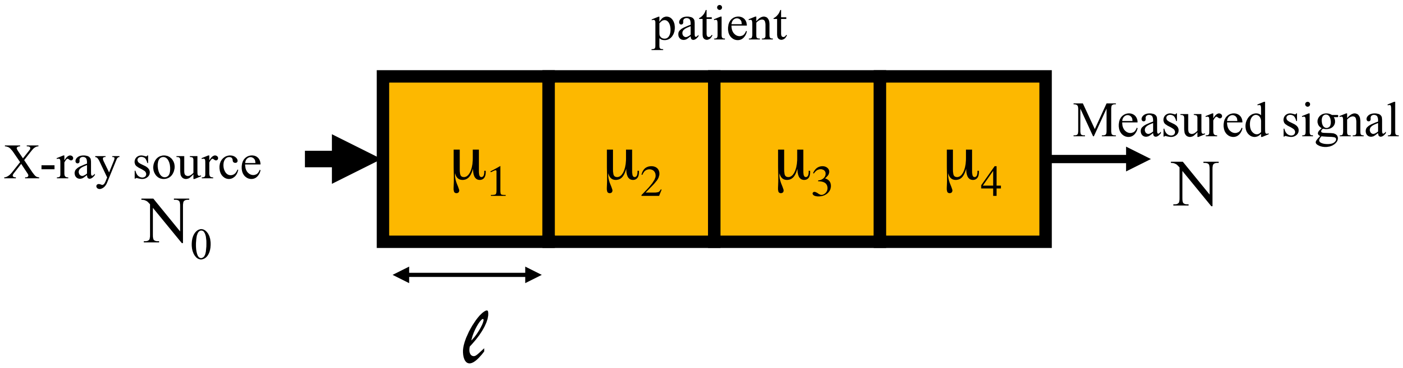
$$N = N_0 \exp(-\mu\ell) \exp(-\mu\ell) \exp(-\mu\ell) \exp(-\mu\ell)$$
$$= N_0 \exp(-\mu\ell - \mu\ell - \mu\ell - \mu\ell) = N_0 \exp(-4\mu\ell)$$

Modelling ET measurements

- Attenuation of an X-ray source in a non-uniform medium



- Discrete expression:

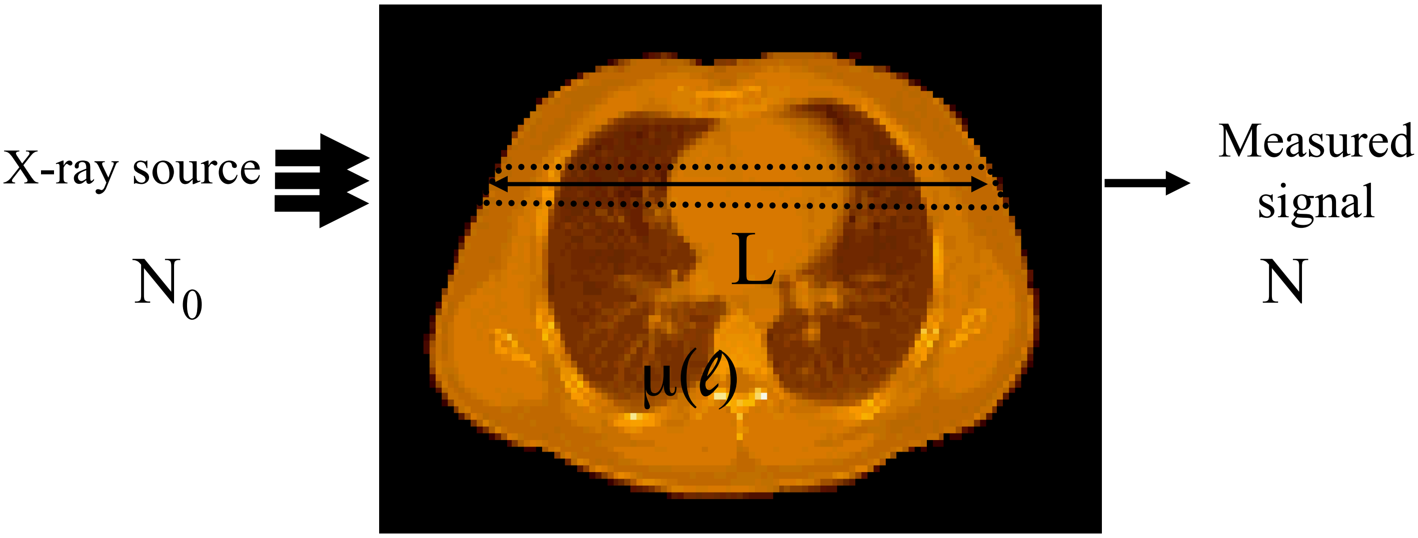


$$N = N_0 \exp[-\mu_1\ell - \mu_2\ell - \mu_3\ell - \mu_4\ell)]$$
$$= N_0 \exp[-(\mu_1+\mu_2+\mu_3+\mu_4)\ell]$$

$$N = N_0 \exp(- \int_0^L \mu(\ell)\, d\ell)$$

Problem to be solved

- Find function $\mu(\ell)$, which is the map of attenuation coefficients μ in the medium of interest



X-ray source

N_0

Measured signal

N

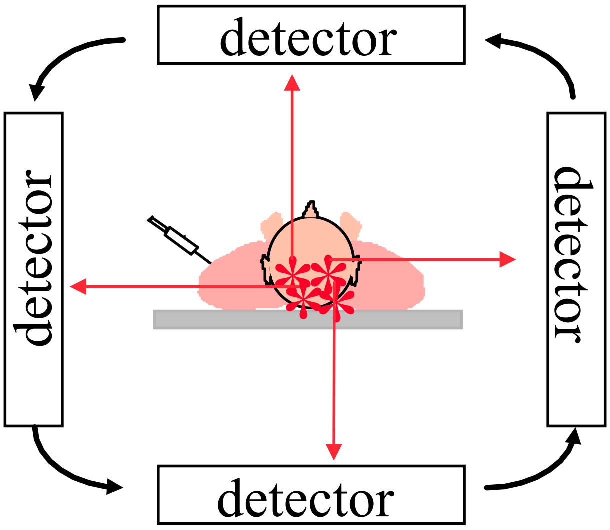
$$N = N_0 \exp\left(-\int_0^L \mu(\ell)\, d\ell\right)$$

$$\ln \frac{N_0}{N} = \int_0^L \mu(\ell)\, d\ell$$

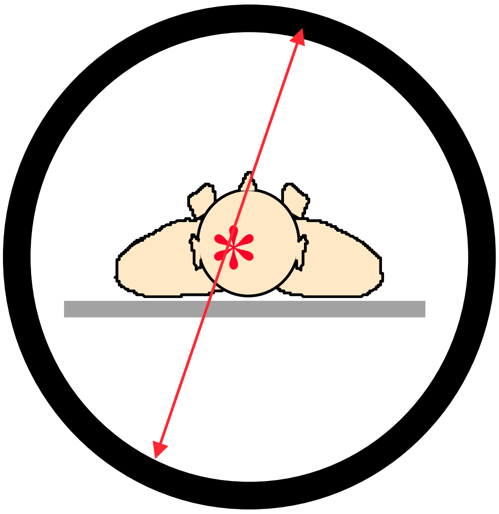
… from integral measurements

Emission tomography devices

- Source γ ou β+ within the patient



SPECT = single photon computed emission tomography

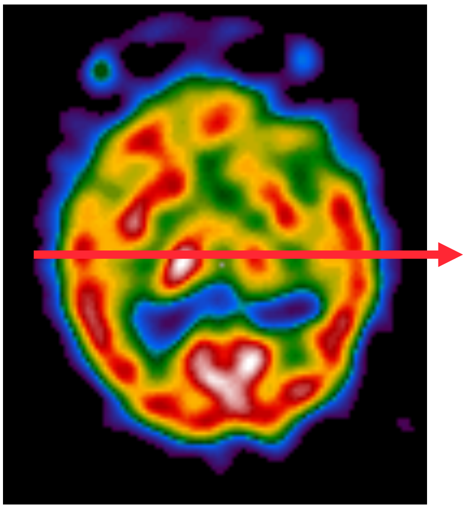


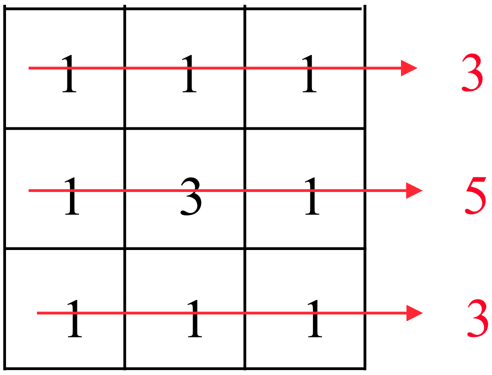
PET = positron emission tomography

Give information regarding the spatial distribution of the source in the body

Emission tomography: mesureaments

- If no attenuation : sum (=integral) of activity along projection lines





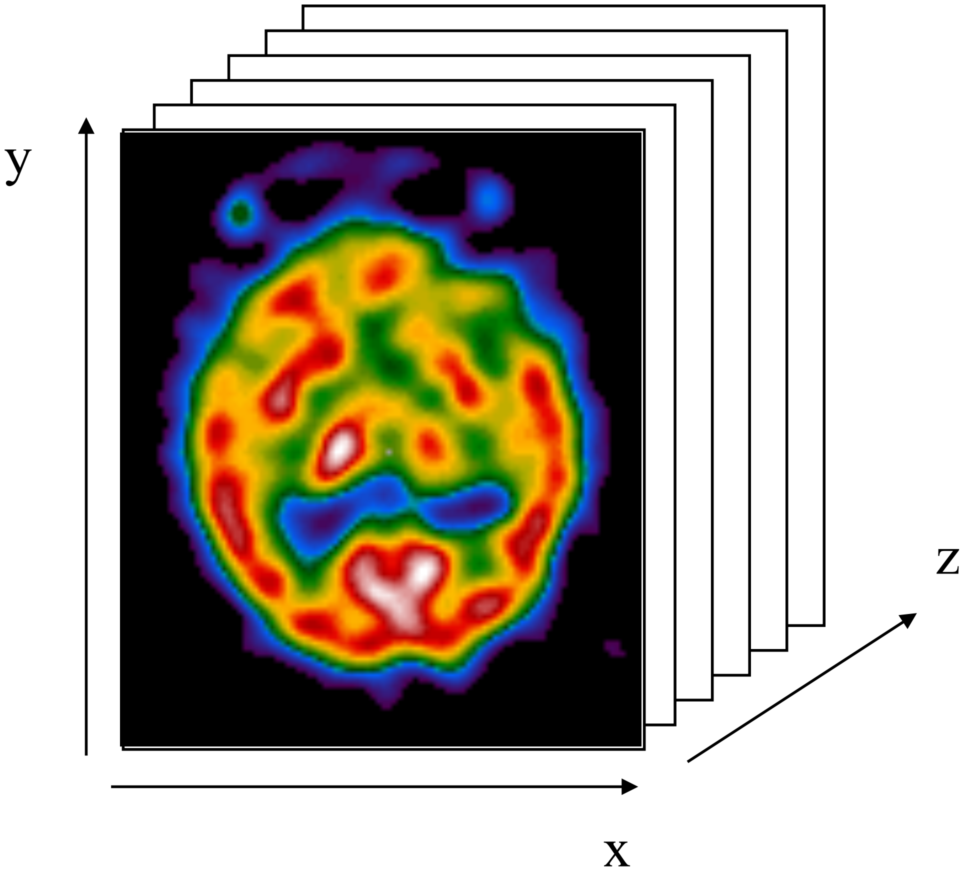
1	1	1	3
1	3	1	5
1	1	1	3

$$N = a_1 + a_2 + a_3$$

$$N = \int_0^D f(\ell)\, d\ell$$

Problem to be solved

- 3D mapping of the activity concentration within the body



In summary

Tomography: estimating the 3D distribution of a parameter of interest based on 2D projections

- Transmission tomography

Parameter of interest = μ attenuation coefficient

- Emission tomography

Parameter of interest = radioactivity map = emission map

Mathematical formalism

Measurements are always integral values (in Emission and Transmission Tomography)

$$\ln \frac{N_0}{N} = \int_0^L \mu(\ell)\, d\ell$$

Known (measured) To be estimated

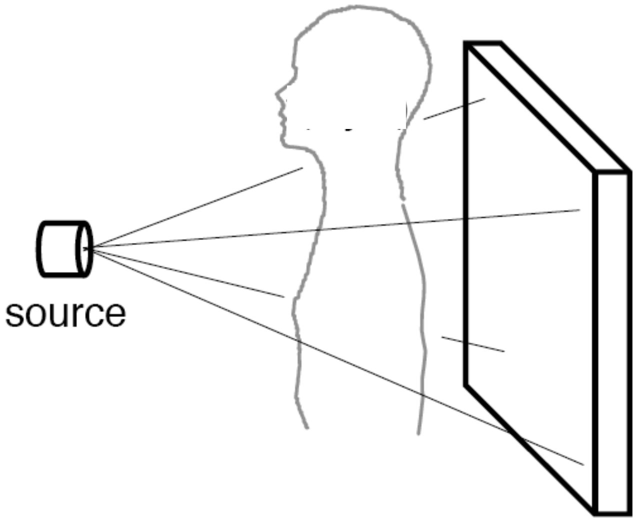
$$N = \int_0^D f(\ell)\, d\ell$$

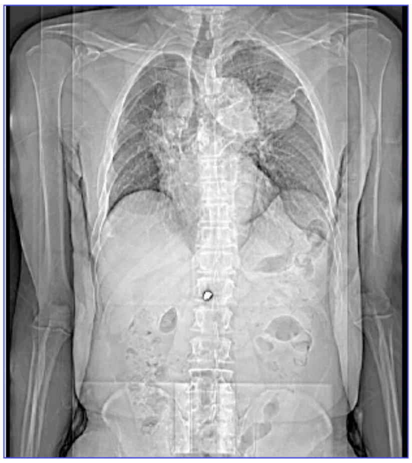
Known (measured) To be estimated

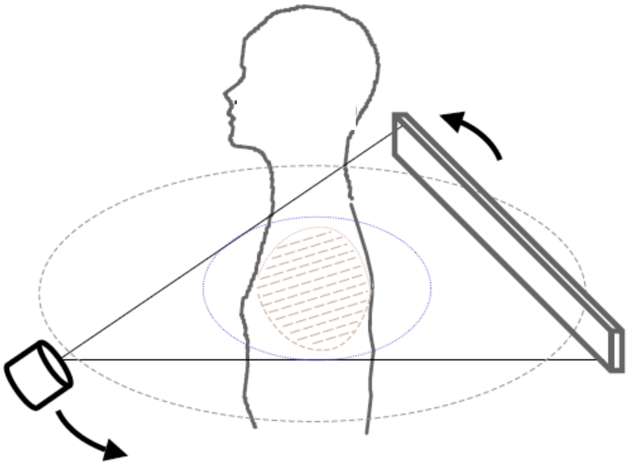
The reconstruction tomography problem obeys the
same formalism
in emission and transmission tomography

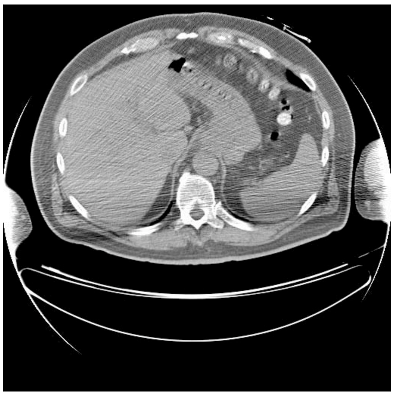
Why is tomography useful? (1)

- Provides volumetric information









The depth of a lesion can be determined

Why is tomography useful? (2)

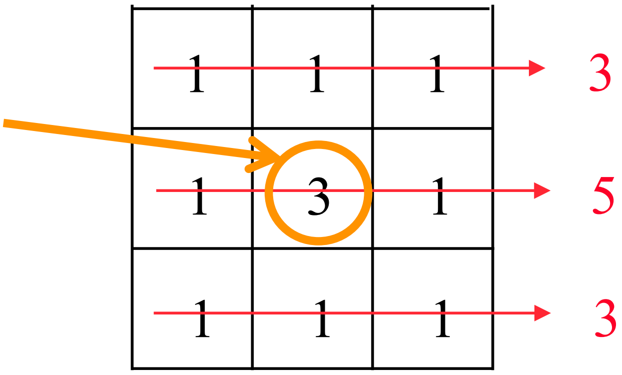
- Increases image contrast

Contrast = (signal of interest – background signal)/ bkgd signal

This is a definition of contrast, there are others

Example in emission tomography

This is supposed to be the lesion



1	1	1	→ 3
1	3	1	→ 5
1	1	1	→ 3

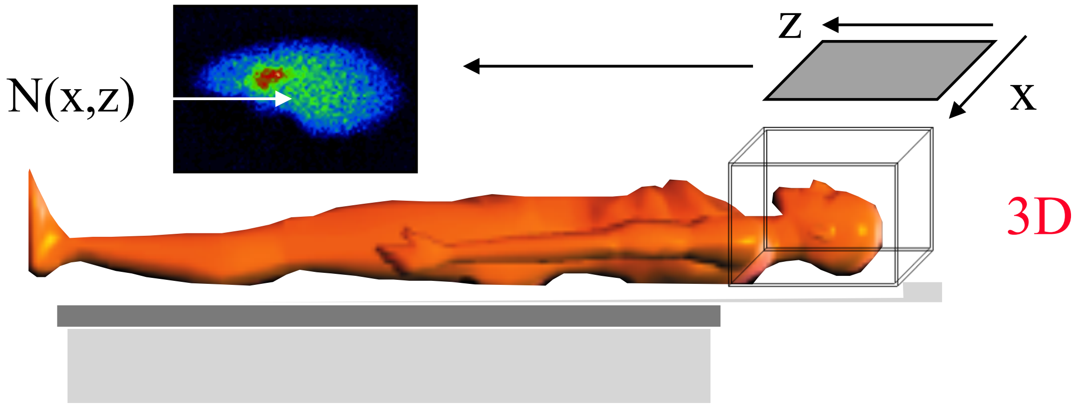
Contrast in the projections : (5-3)/3 = 0.66
Contrast in the slice (cross section): (3-1)/1 = 2

Lesions will be easier to detect in reconstructed slices !

Reconstruction problem in 3D

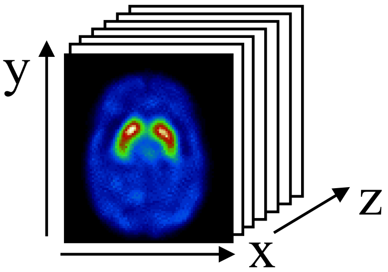
- Measurement of a set of 2D projections

projection θ ← detector plan at position θ



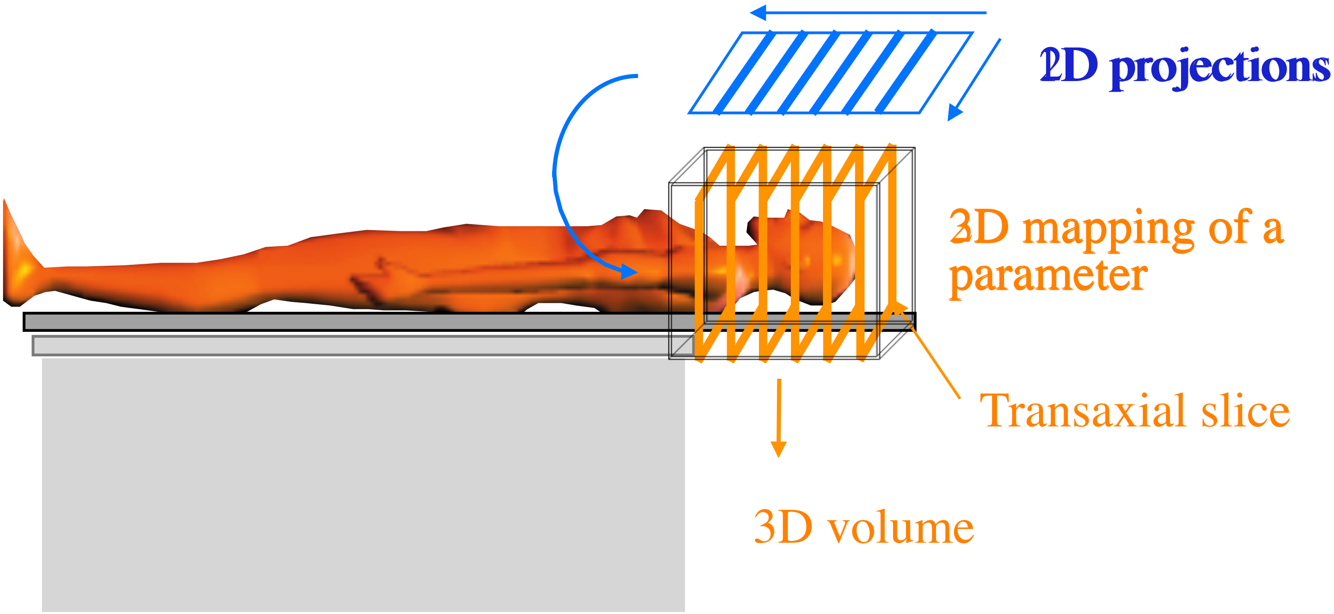
3D volume of interest

➡ Tomographic reconstruction



Factorization of the reconstruction problem

A 3D volume can be seen as a stack of 2D images

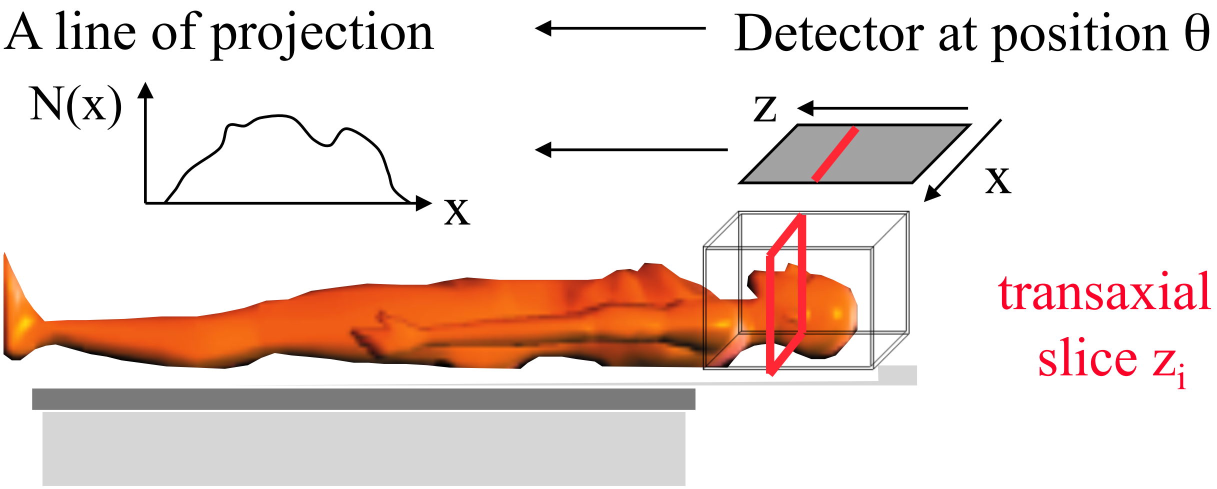


3D volume reconstructed
from a set of 2D images

So what has to be understood is
how to reconstruct a 2D slice from a set of 1 D projections

2D formalism

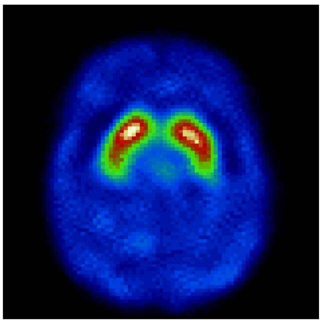
- A set of 1D projections



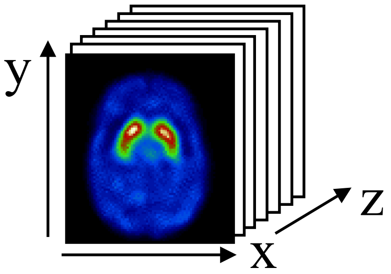
A line of projection ← Detector at position θ

transaxial slice z_i

➡ reconstruction of a 2D signal (z_i slice)

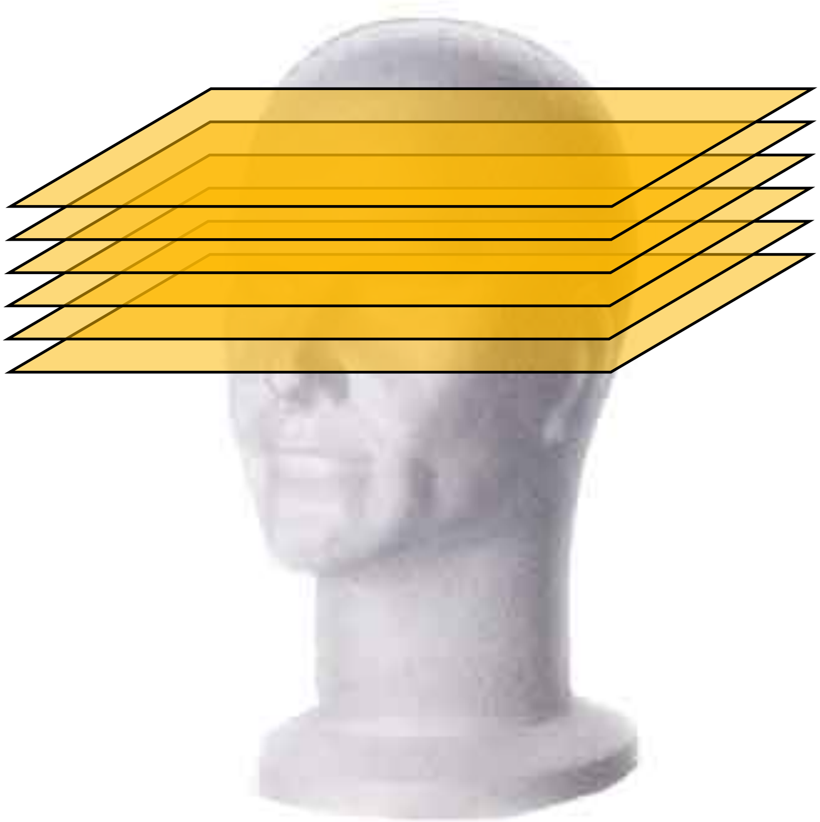


Set of slices z_i = volumec of interest



Tomographic reconstruction in general

… is estimating a 3D volume by independent reconstruction of a set of 2D slices



Direct reconstruction of a 3D volume is actually called "Fully 3D reconstruction"

Why is it so difficult?

La leçon difficile, William Bouguereau (1825 - 1905)

1) Because the solution is non unique

- No unique solution : there are ALWAYS several signal distributions compatible with the finite number of measured projections

1 projection : several possible solutions

projection direction

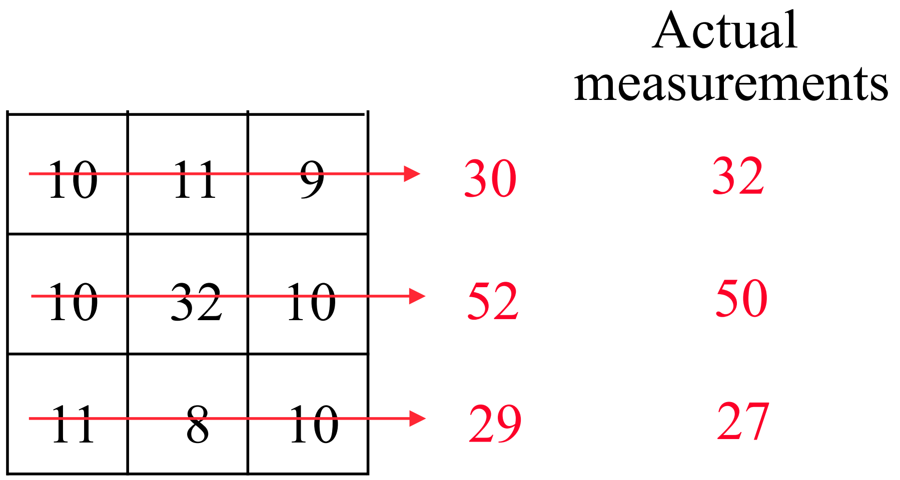
2 projections : several possible solutions

projection direction

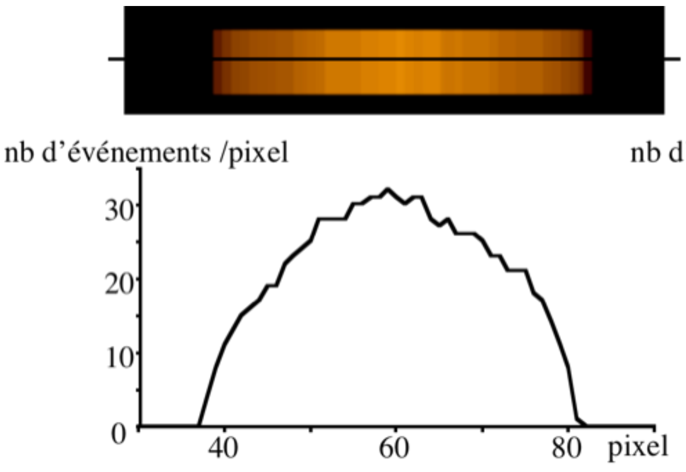
➡ A unique solution would exist only for an infinite number of noiseless continuous projections

2) Projections are noisy

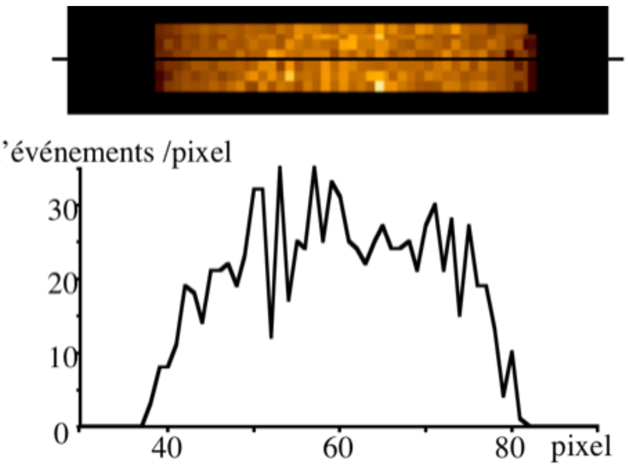
- No exact solution, because the measurements are corrupted by noise



				Actual measurements
10	11	9	30	32
10	32	10	52	50
11	8	10	29	27



ideal projection



noisy projection

An ill-posed inverse problem

inverse problems

- Inverse problem :
We have measurements, we want to determine which signal produced the detected measurements

- Ill-posed problem :
The solution is unstable (sampling + noise) : two different measurements can lead to significantly different solutions

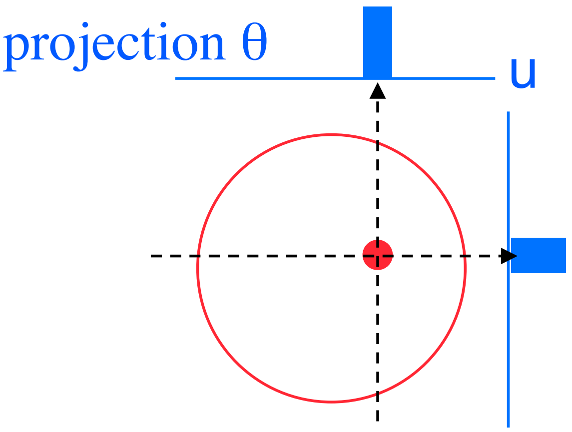
Basic concepts

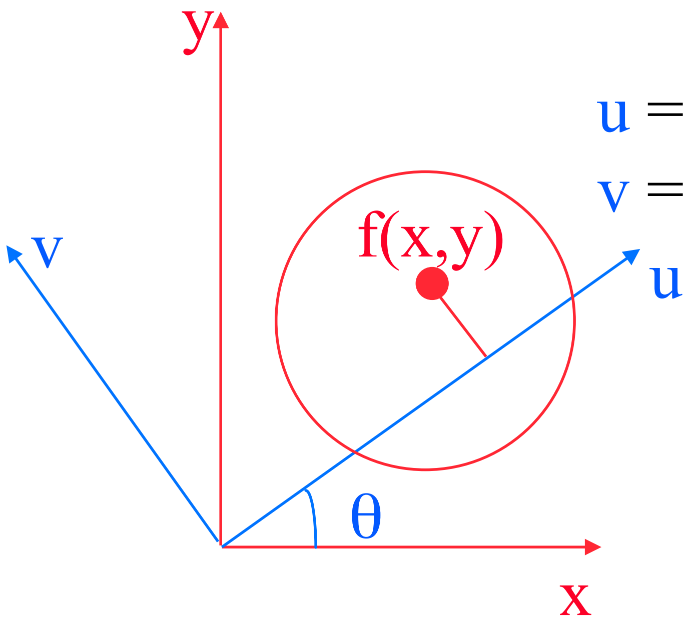
Seminal work

1887-1956

1917 : Johann Radon : "About the determination of functions from their integral functions in certain directions", Math. Phys. Klass.

Projection operator : continuous formalism





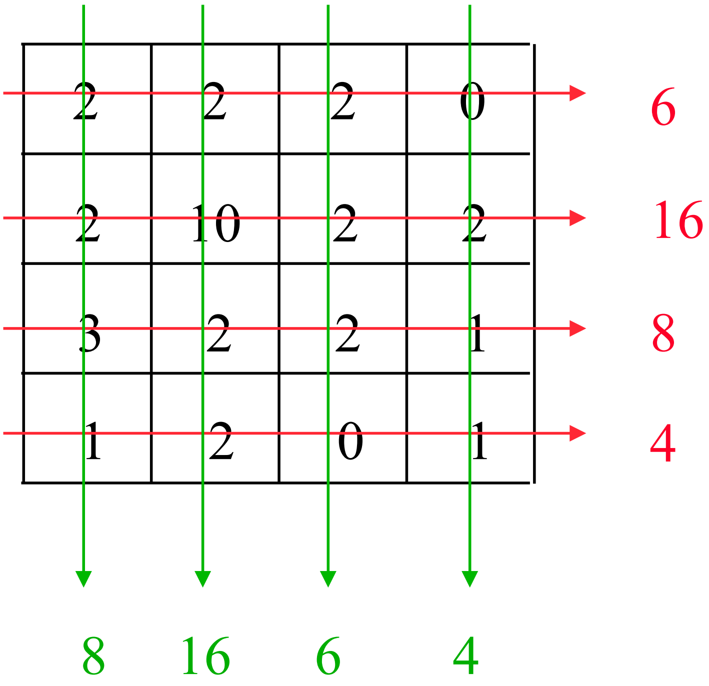
$$u = x \cos\theta + y \sin\theta$$

$$v = -x \sin\theta + y \cos\theta$$

$$p(u,\theta) = \int_{-\infty}^{+\infty} f(x,y)\, dv$$

Projection operator : discrete formalism

- Calculate the two 2 projections along the green and red directions of this activity distribution



2	2	2	0	6
2	10	2	2	16
3	2	2	1	8
1	2	0	1	4
8	16	6	4	

Radon transform

$$p(u,\theta) = \int_{-\infty}^{+\infty} f(x,y)\, dv$$

Set of projections for θ = [0, π]
= Radon transform of f(x,y)

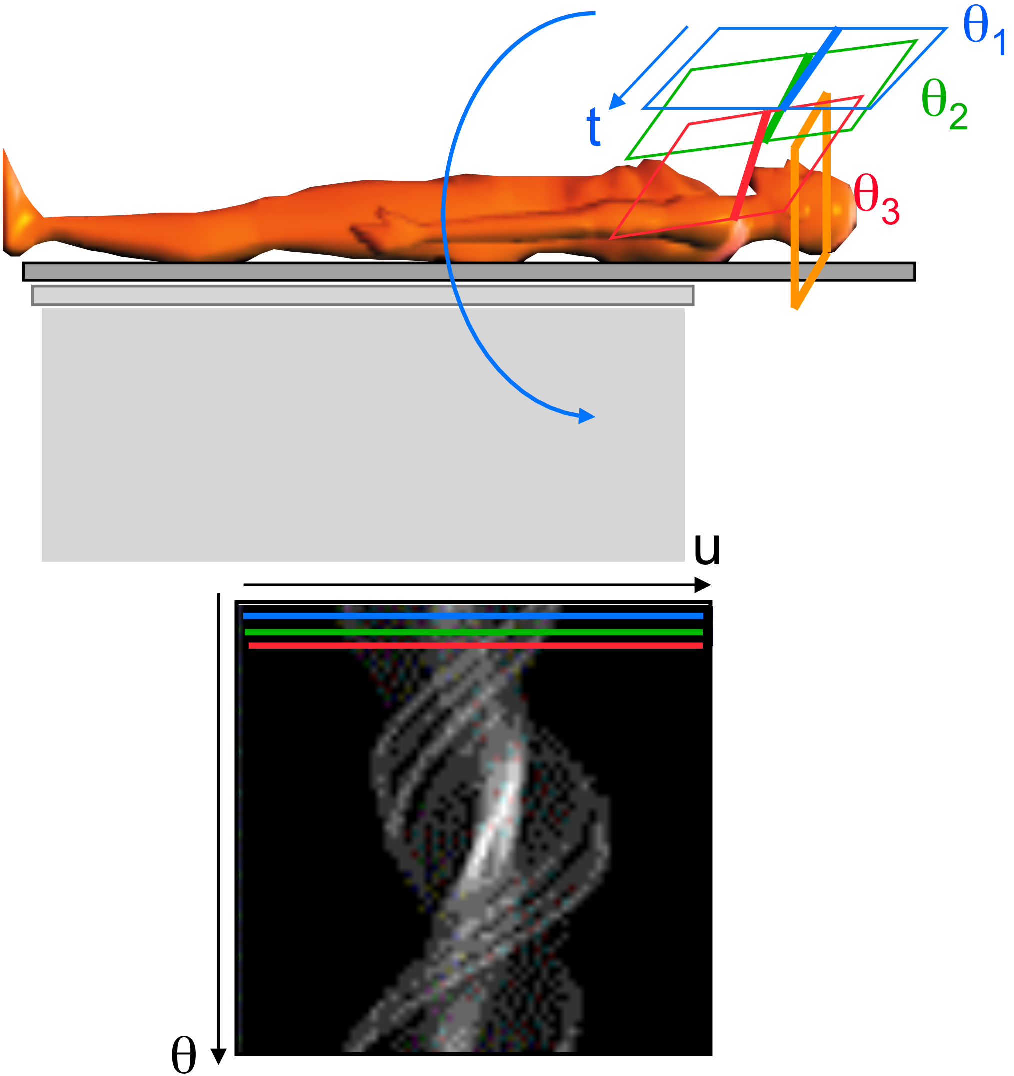
$f(x,y) \longrightarrow p(u,\theta)$

Spatial domain Radon domain

Tomographic reconstruction :
Inversion of the Radon transform, i.e.,
Estimation of f(x,y) from p(u,θ)

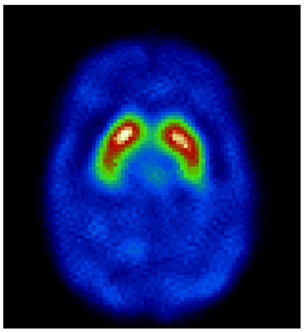
Sinogram

Sinogram = signal from slice z_i recorded at different angles θ



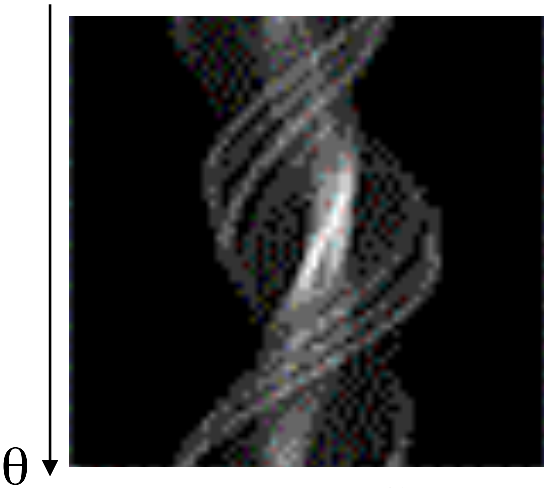
sinogram corresponding to slice z_i

$\xrightarrow{\text{tomographic reconstruction}}$ slice z_i



Sinogram and projections

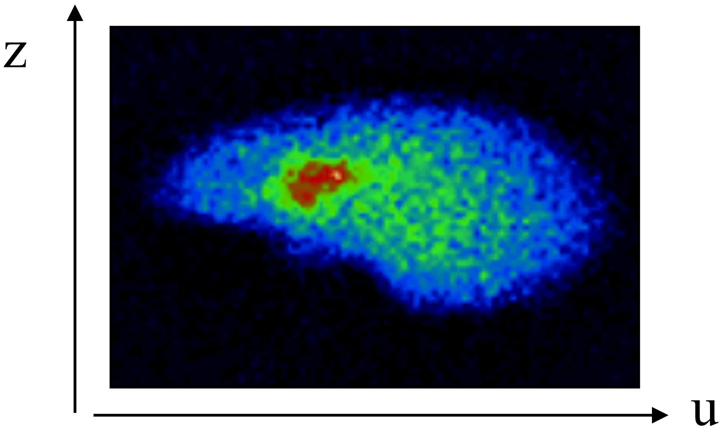
Sinograms and projections contain the same information but stored differently



sinogram corresponding to slice z_i

A sinogram: all information pertaining to a given slice

A single sinogram is sufficient to reconstruct a slice



projection corresponding to angle θ

A projection : information regarding all slices for a given projection angle. With a single projection, it is impossible to reconstruct a slice

Test

We record 64 projections of 128 pixels (along the axial direction) x 256 pixels

- How many transaxial slices can be reconstructed without interpolation ?

128

- How many sinograms can we derive from the projections?

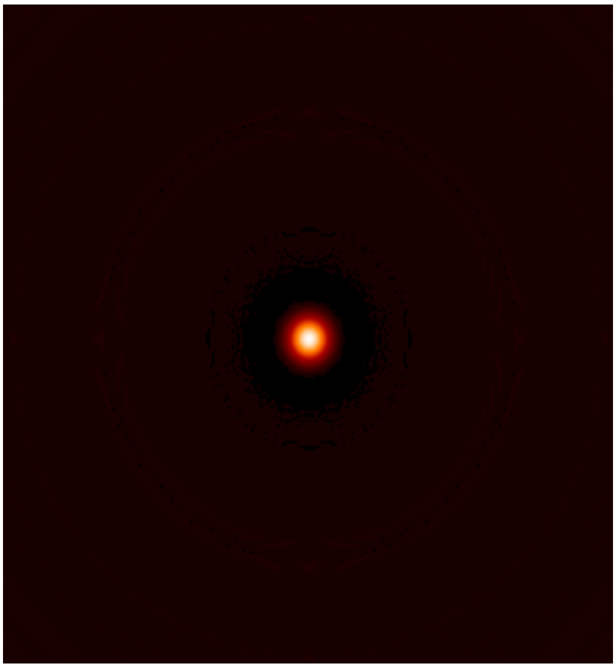
128

- What are the sinogram dimensions (number of rows and number of columns) ?

64 rows et 256 columns

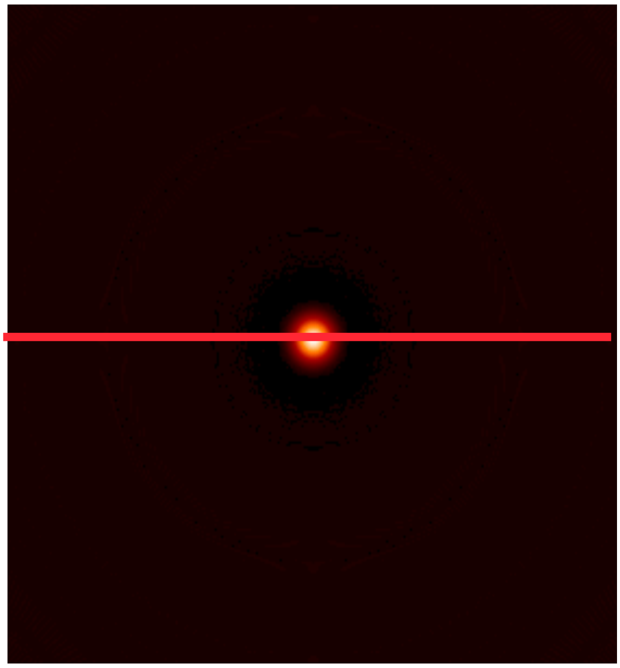
Test

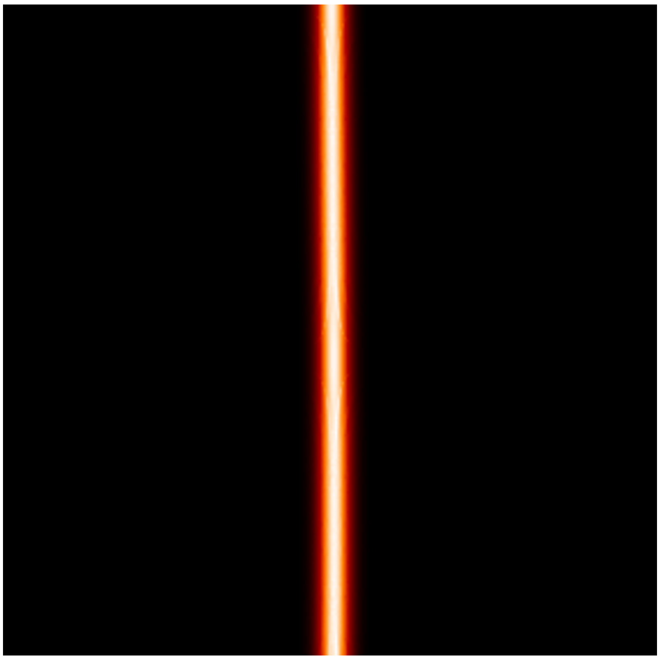
Emission tomography:
Is it a projection or a sinogram?



Test

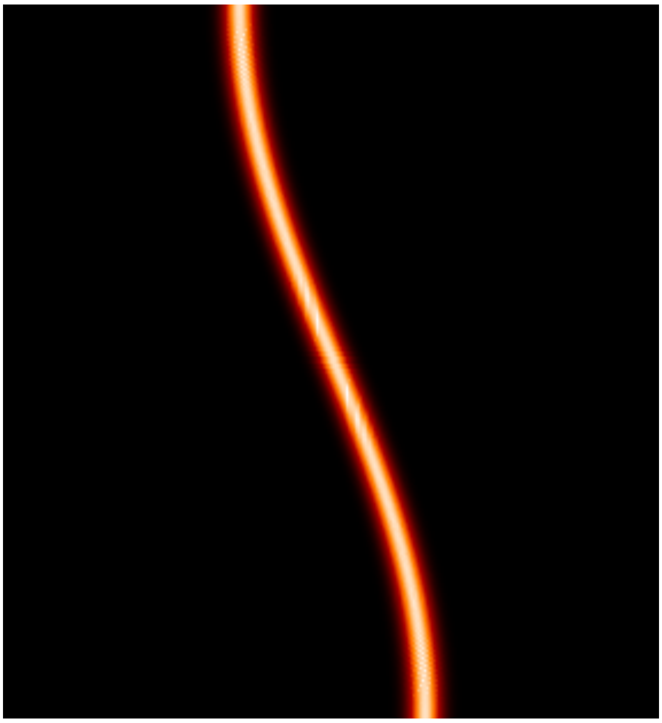
Emission tomography:
If all projections are identical to this one, what is the sinogram corresponding to the slice located at the red line position?

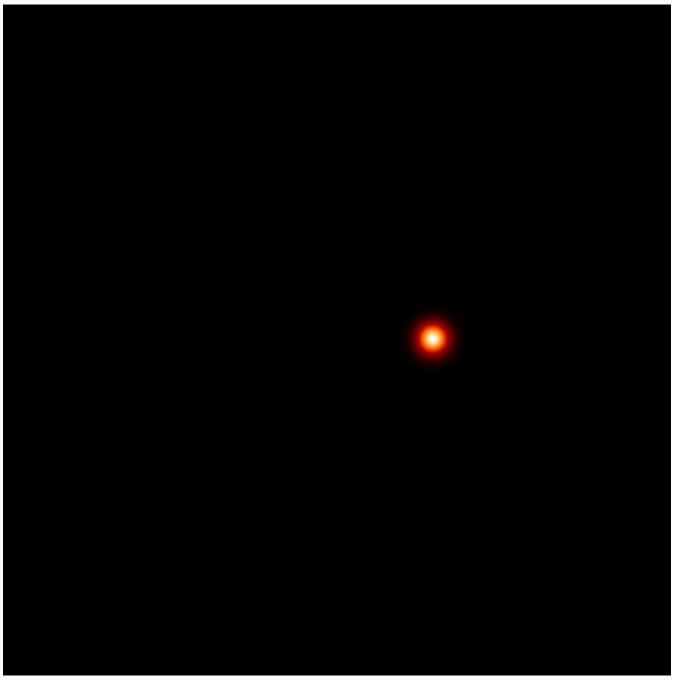




Test

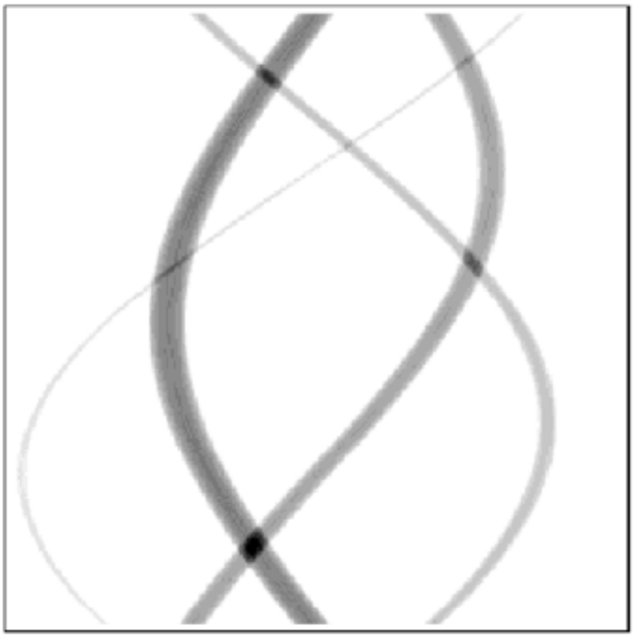
Emission tomography:
What is the reconstructed signal corresponding to this sinogram?

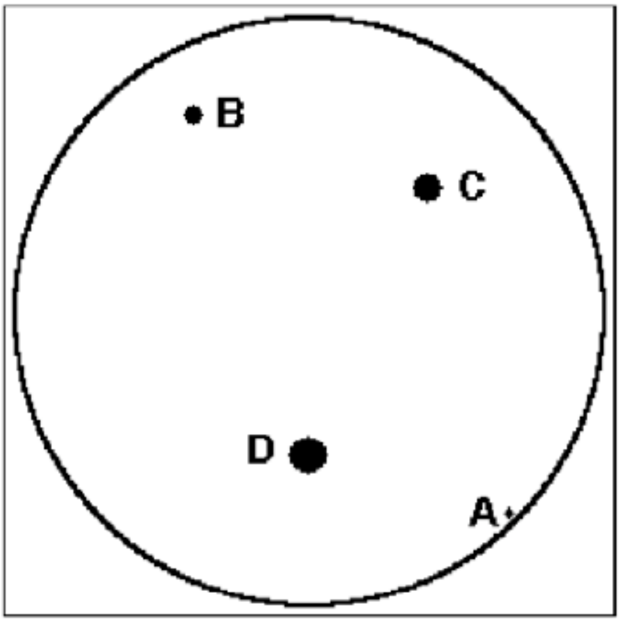




Test

Emission tomography:
What is the reconstructed signal corresponding to this sinogram?





Summary

- Introduction
  - What is tomography?
  - Transmission tomography
  - Emission tomography
  - Why is tomographic reconstruction so difficult?

- Basic concepts
  - Projection
  - Radon transform
  - Sinogram

- Tomography consists in estimating cross section images from measured projections

- To perform tomography, several views of the object of interest recorded at different angles are required

- A projection element is the integral of the signal along a projection line, a projection is the set of projection elements recorded at a given angle, the set of all projections is the Radon transform of the object of interest

- In a projection, different signals overlap and contrast is reduced

- Tomographic reconstruction consists in estimating the signal of interest that yielded the measured projections using a mathematical algorithm. It is an ill-posed problem.

Two approaches for tomographic reconstruction

- Analytical methods

$$R[f(x,y)] = \int_0^\pi p(u,\theta)\, d\theta$$

- Discrete or iterative methods

$$p = R\, f$$

Analytical methods: introduction

- Consist in an analytical inversion of the Radon transform = solving integral equations

- The tomographic reconstruction problem is expressed using a continuous formalism

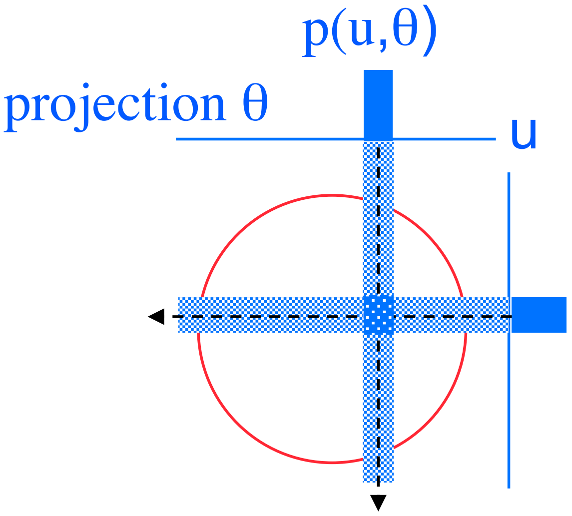
- THE analyical method that is always used

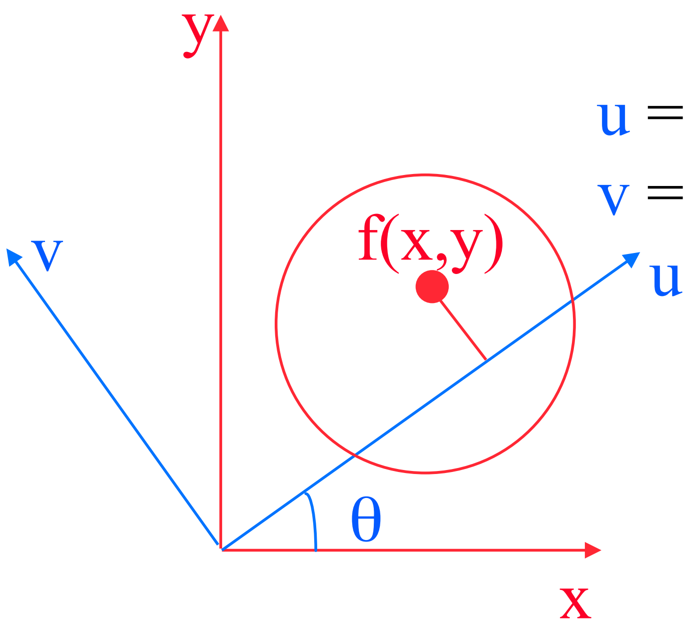
FBP : Filtered BackProjection

- FBP is FAST

- FBP is available on all commercial scanners (X-ray scanners, SPECT and PET devices)

Backrojection operator : continuous formalism





$u = x \cos\theta + y \sin\theta$

$v = -x \sin\theta + y \cos\theta$

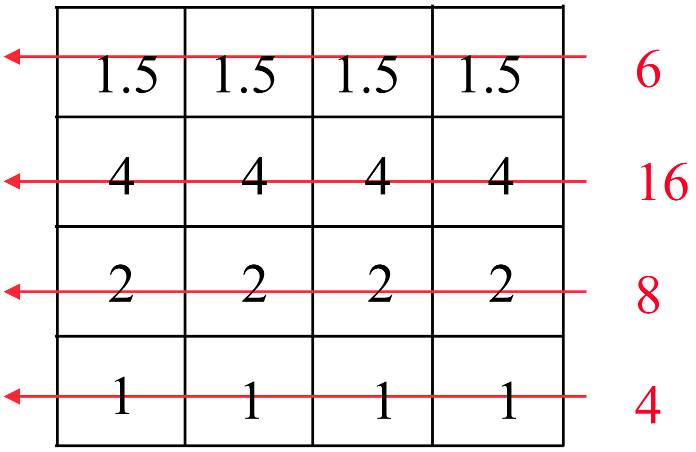
$$f^*(x,y) = \int_0^{\pi} p(u,\theta)\, d\theta$$

Beware: backprojection does NOT invert the Radon transform

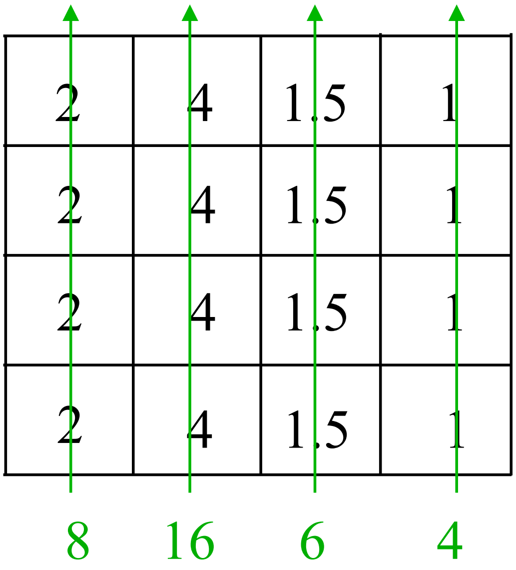
Backrojection operator : discrete formalism

- Calculate the backprojection of the measured green and red projections

				6
				16
				8
				4
8	16	6	4	

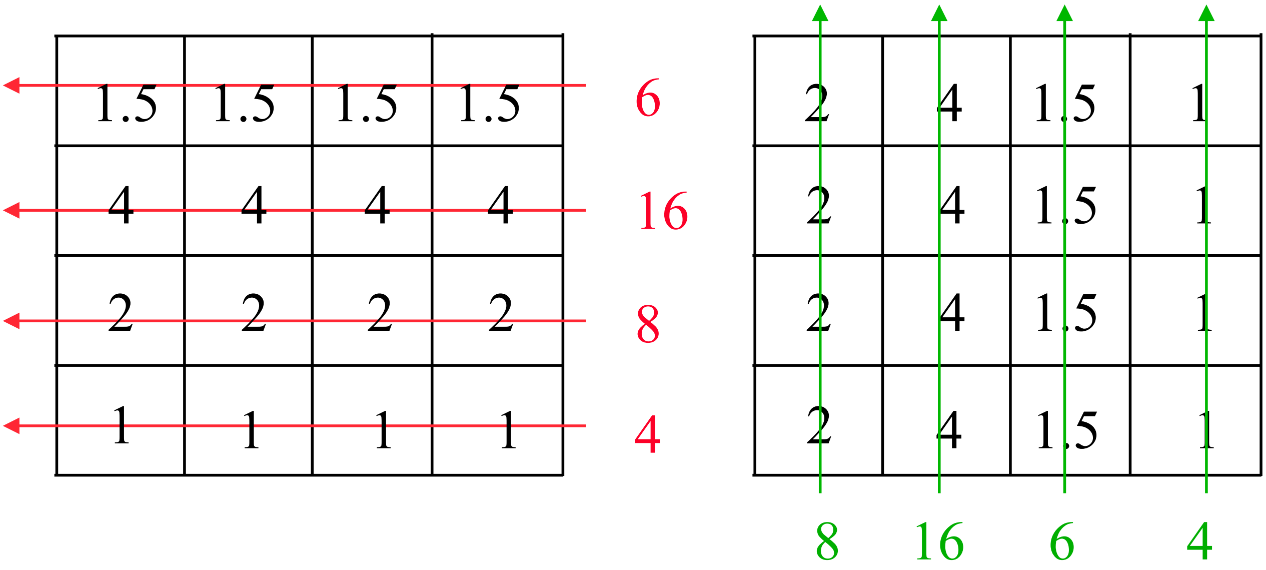


1.5	1.5	1.5	1.5	6
4	4	4	4	16
2	2	2	2	8
1	1	1	1	4



2	4	1.5	1
2	4	1.5	1
2	4	1.5	1
2	4	1.5	1
8	16	6	4

Backrojection operator : discrete formalism



1.5	1.5	1.5	1.5	6
4	4	4	4	16
2	2	2	2	8
1	1	1	1	4

2	4	1.5	1
2	4	1.5	1
2	4	1.5	1
2	4	1.5	1
8	16	6	4

Mean:

tumor / bckgd ratio
= 4 / 1.5 = 2.6

1.75	2.75	1.5	1.25
3	4	2.75	2.5
2	3	1.75	1.5
1.5	2.5	1.25	1

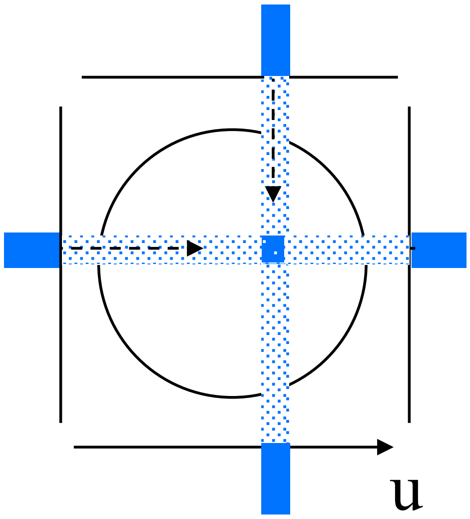
Original image:

tumor / bckgd ratio
= 10 / 1 = 10

2	2	2	0
2	10	2	2
3	2	2	1
1	2	0	1

Backprojection does NOT invert the Radon transform

Backprojection limitations

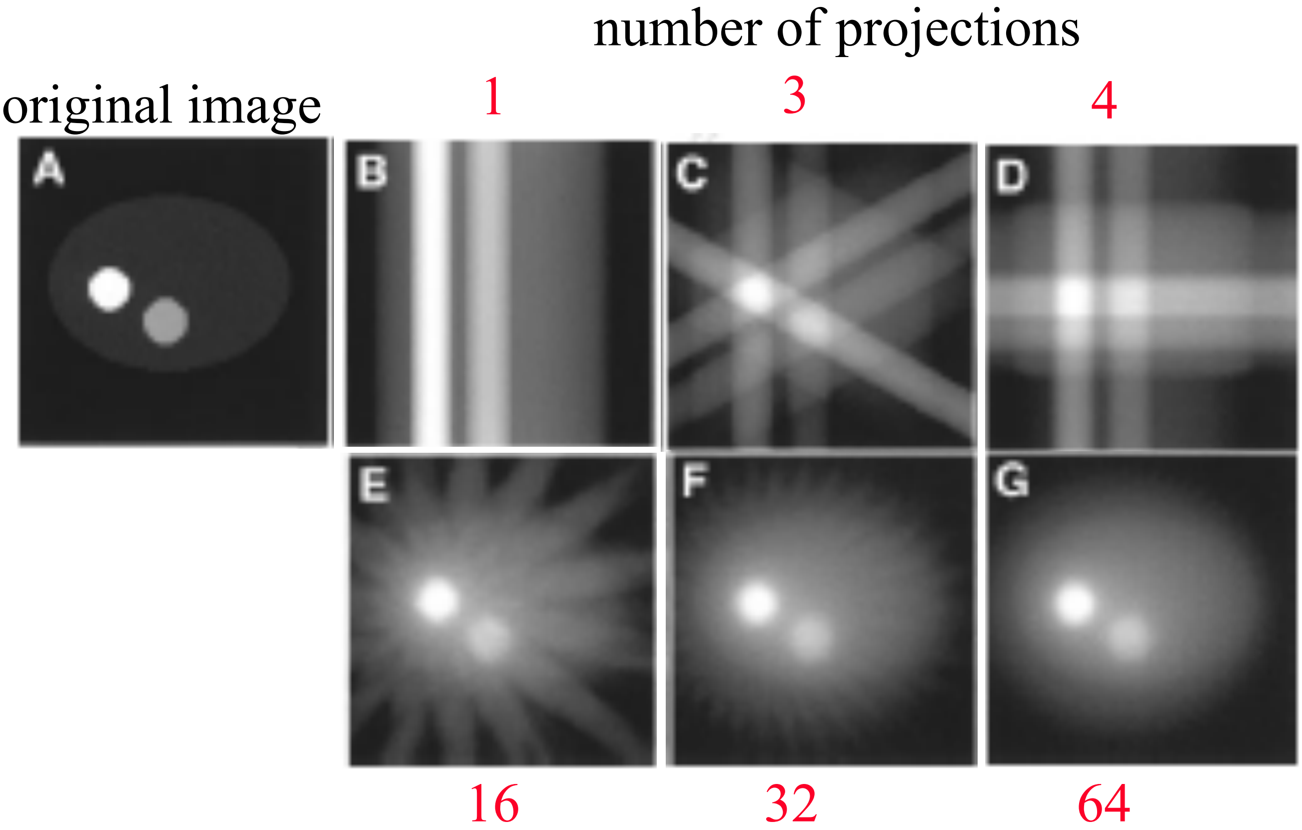


$$f^*(x,y) = \int_0^\pi p(u,\theta)\, d\theta$$

backprojection

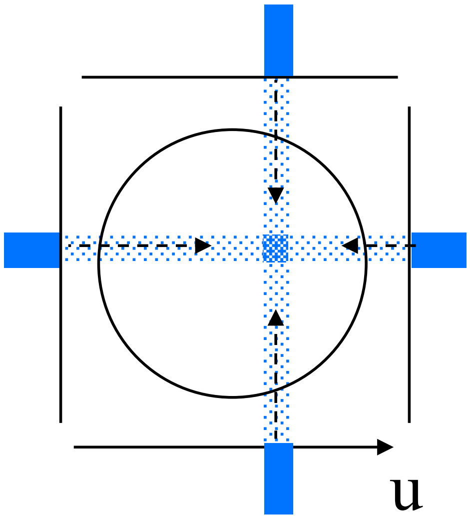
➡ streak artefacts

due to the limited number of projections



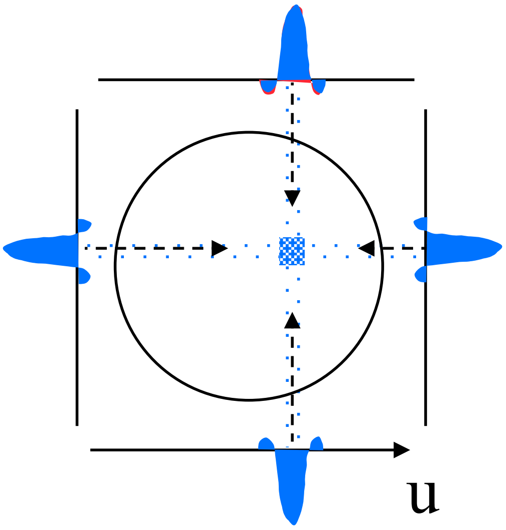
Backprojection does NOT invert the Radon transform

Filtered backprojection: principle



$$f^*(x,y) = \int_0^\pi p(u,\theta)\, d\theta$$

backprojection



$$f^*(x,y) = \int_0^\pi p(u,\theta)\, d\theta$$

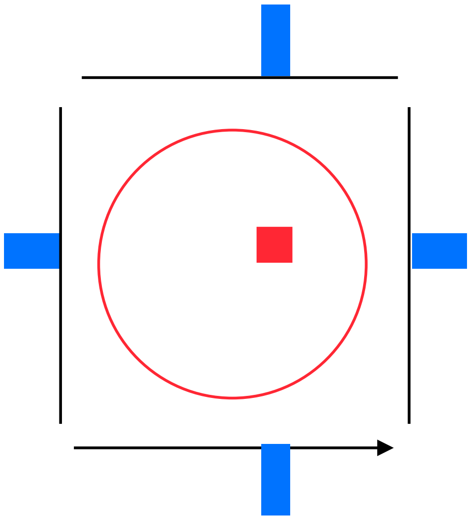
filtered projection

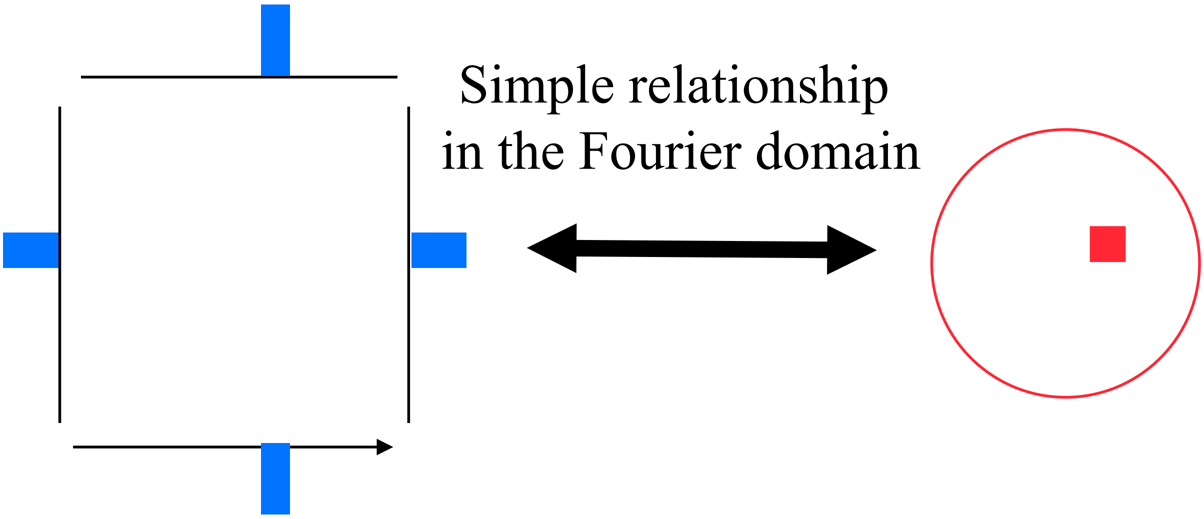
filtered backprojection
reduction of streak artefacts
Exact inversion of the Radon transform !

Which filter?

The filter that makes it possible to accurately invert the Radon transform can be theoretically derived using the central slice theorem

This theorem establishes the relationship between the projections and the object in the Fourier domain



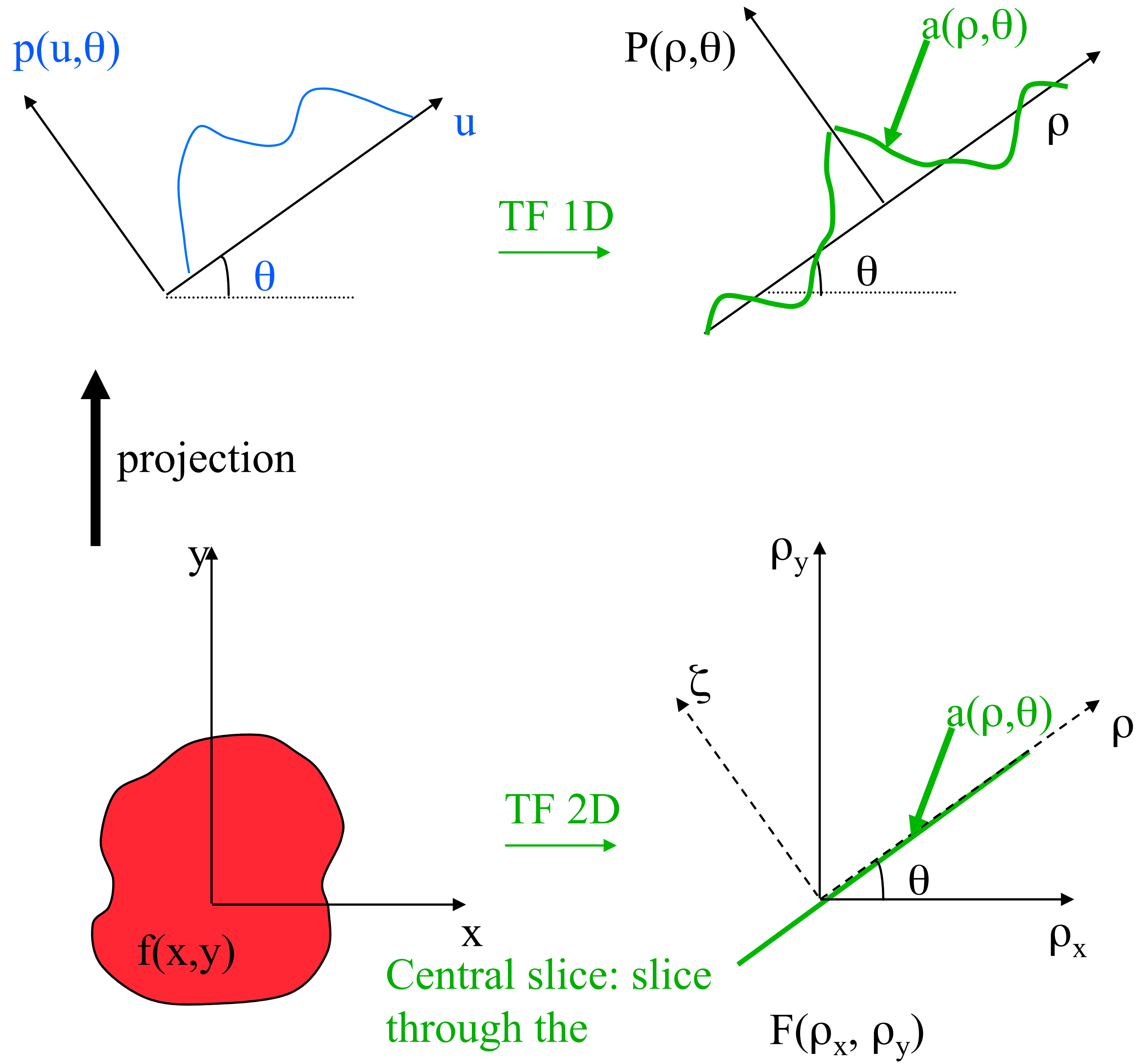


Simple relationship in the Fourier domain

Central slice theorem

1D FT of a projection with respect to u
=
2D FT of the signal to be reconstructed

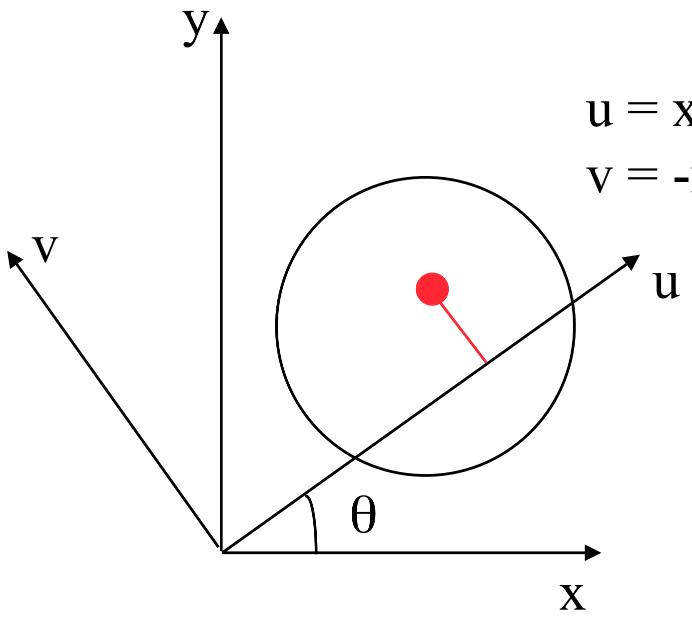
$$P(\rho,\theta) = F(\rho_x, \rho_y)\Big|_{\zeta=0}$$



Central slice: slice through the referential origin

Central slice theorem: demonstration

$$p(u,\theta) = \int_{-\infty}^{+\infty} f(x,y)\, dv \xrightarrow{\text{1D FT}} P(\rho,\theta) = \int_{-\infty}^{+\infty} p(u,\theta) e^{-i2\pi\rho u}\, du$$



$u = x \cos\theta + y \sin\theta$
$v = -x \sin\theta + y \cos\theta$

$\rho_x = \rho \cos\theta$
$\rho_y = \rho \sin\theta$
$du.dv = dx.dy$

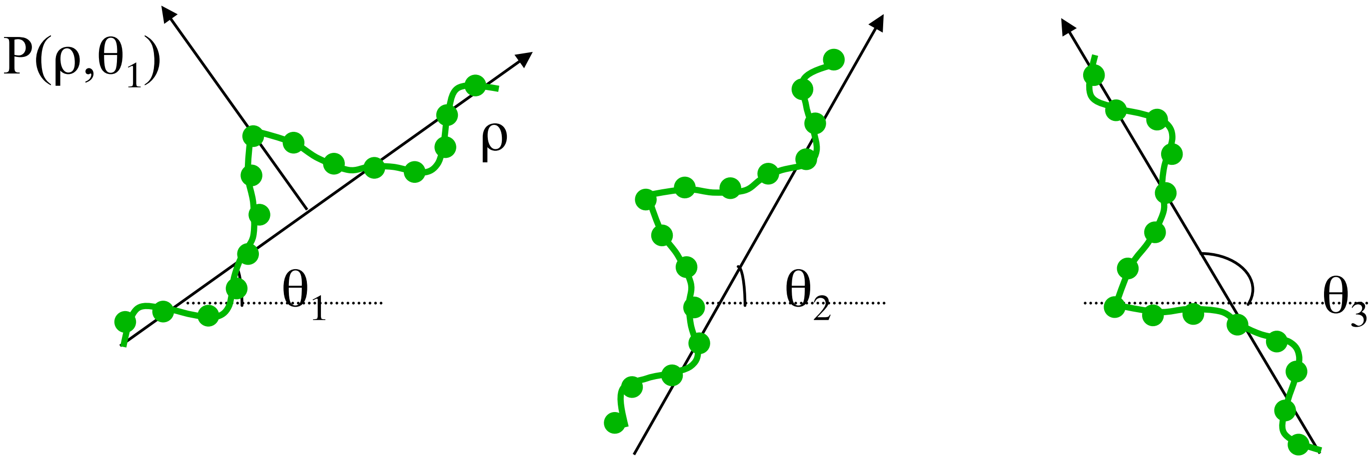
Change of variables : $(u,v) \longrightarrow (x,y)$

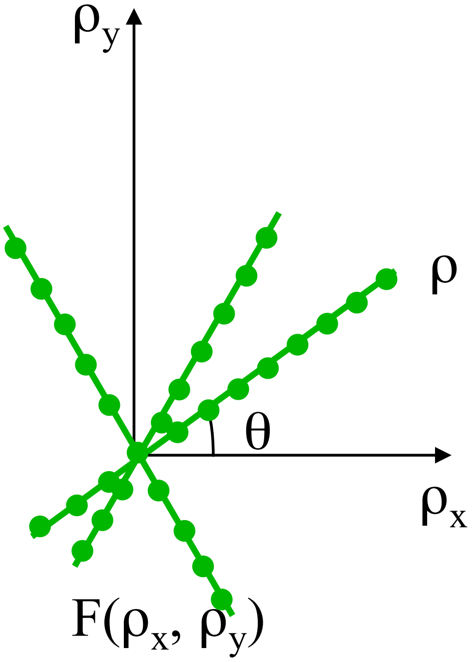
$$P(\rho,\theta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y) e^{-i2\pi\rho u}\, du\, dv \overset{\downarrow}{=} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y) e^{-i2\pi(x\rho_x + y\rho_y)}\, dx\, dy$$

1D FT of a projection with respect to u
=
2D FT of the signal to be reconstructed

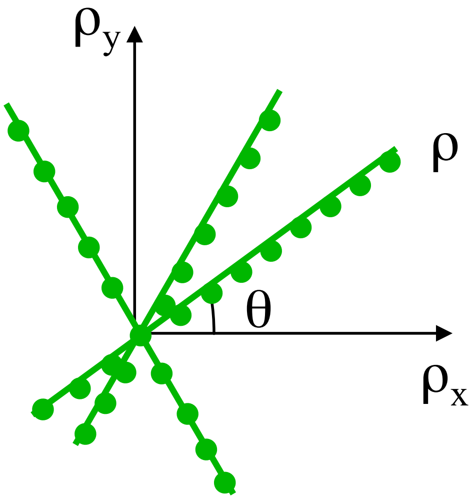
Filtered backprojection: principle

If $P(\rho,\theta)$ is known for all angles θ between 0 and π, the FT of the object can be reconstructed, hence the object can be estimated

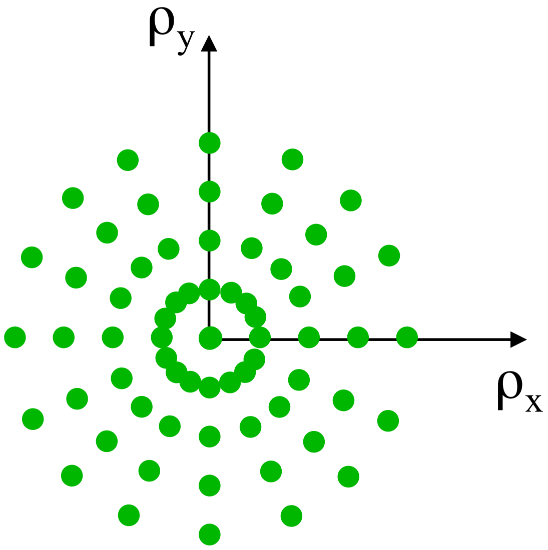




Filtered backprojection: need for a filter



$F(\rho_x, \rho_y)$



Points are irregularly sampled in the Fourier space : the density of points is proportional to $1/|\rho|$: low frequency signal is therefore weighted more. This introduces a blur in the reconstructed images when using backprojection only. A correction (filter) for this irregular sampling is needed to avoid that blur.

Filtered backprojection: demonstration

$$\boxed{P(\rho,\theta) = F(\rho_x, \rho_y)}$$

FT^{-1}

$$f(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\rho_x,\rho_y)e^{i2\pi\,(x\rho_x + y\rho_y)}\, d\rho_x\, d\rho_y$$

CST

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P(\rho,\theta)e^{i2\pi(x\rho_x + y\rho_y)}\, d\rho_x\, d\rho_y$$

$\rho_x = \rho \cos \theta$
$\rho_y = \rho \sin \theta$
$\rho = (\rho_x{}^2 + \rho_y{}^2)^{1/2}$
$d\rho_x.d\rho_y = \rho.d\rho.d\theta$
$u = x \cos\theta + y \sin\theta$

Change of variable : $(\rho_x, \rho_y) \rightarrow (\rho, \theta)$

$$= \int_{0}^{\pi}\int_{-\infty}^{+\infty} P(\rho,\theta)\, |\rho| e^{i2\pi\rho u}\, d\rho\, d\theta$$

$$= \int_{0}^{\pi} p'(u,\theta)\, d\theta \quad \text{with } p'(u,\theta) = \int_{-\infty}^{+\infty} P(\rho,\theta)\, |\rho| e^{i2\pi\rho u}\, d\rho$$

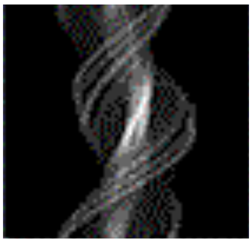
filtered projections

ramp filter

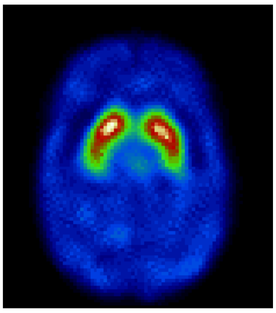
f(x,y) function
=
backprojection of the filtered projections

Filtered backprojection algorithm

$$f(x,y) = \int_{0}^{\pi} p'(u,\theta)\, d\theta \quad \text{with } p'(u,\theta) = \int_{-\infty}^{+\infty} P(\rho,\theta)\, |\rho| e^{i2\pi\rho u}\, d\rho$$



sinogram



reconstructed slice

$$\begin{tikzcd}
p(u,\theta) \arrow[d, "\text{1D Fourier transform}"] & & f(x,y) \\
P(\rho,\theta) \arrow[r, "\text{filtering}"] & |\rho|\, P(\rho,\theta) \arrow[r, "\text{inverse Fourier transform}"] & p'(u,\theta) \arrow[u, "\text{backprojection}"']
\end{tikzcd}$$

Summary

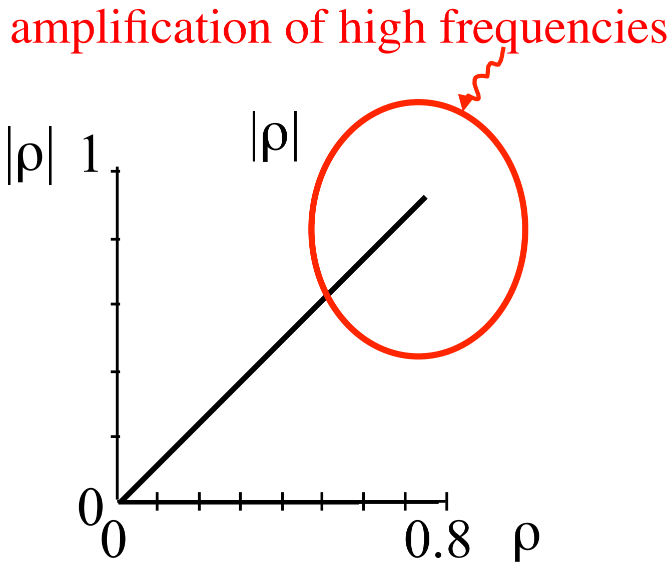
- Analytical reconstruction
    - Principle
    - Central slice theorem
    - Filtered backprojection
    - Filters (next)

- Backprojection is a key ingredient to tomographic reconstruction : this operation redistributes the signal measured in the projection to the image space. Yet, because the spatial domain space and the Fourier space are not sampled identically, backprojection images include low frequency streak artefacts

- An exact inversion of the Radon transform is feasible based on the Central Slice Theorem, accounting for the differences in sampling in the spatial domain space and Fourier space

Why is the ramp filter not sufficient?

$$f(x,y) = \int_0^{\pi} p'(u,\theta)\, d\theta \quad \text{with } p'(u,\theta) = \int_{-\infty}^{+\infty} P(\rho,\theta)\, \underset{\substack{\uparrow \\ \text{ramp filter}}}{|\rho|} e^{i2\pi\rho u}\, d\rho$$



ramp filter

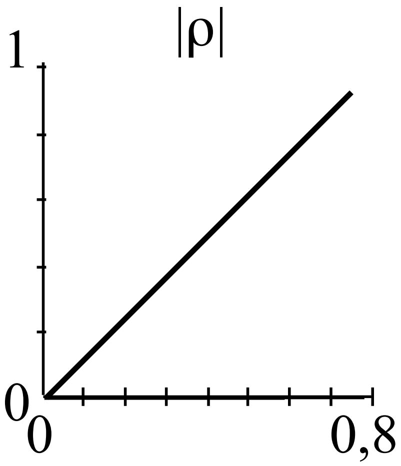
High frequencies = details in the images (high spatial resolution requires high frequency information)

But also noise !

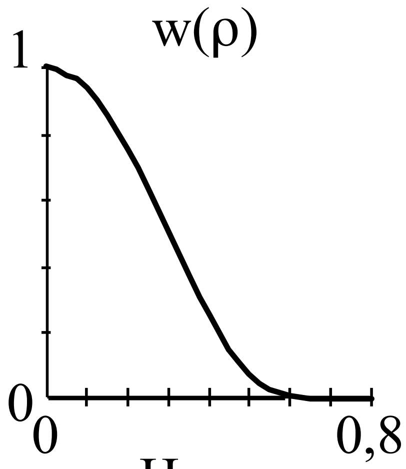
Why is the ramp filter not sufficient?

$$f(x,y) = \int_0^{\pi} p'(u,\theta)\, d\theta \quad \text{with } p'(u,\theta) = \int_{-\infty}^{+\infty} P(\rho,\theta)\, \underset{\substack{\uparrow \\ \text{ramp filter}}}{|\rho|} e^{i2\pi\rho u}\, d\rho$$

$$|\rho| \longrightarrow |\rho| \underset{\substack{\uparrow \\ \text{apodization window}}}{w(\rho)}$$

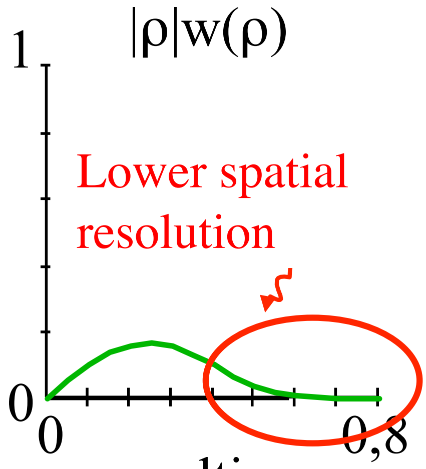


ramp filter



Hann apodization window

→



resulting Hann filter

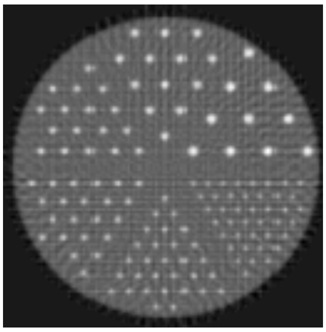
$$w(\rho) = 0.5.(1+\cos\pi\rho/\rho_c) \quad \text{if } \rho < \rho_c$$
$$= 0 \quad \text{if } \rho \geq \rho_c$$

Fourier domain

Usual filters : Hann filter

- ramp filter

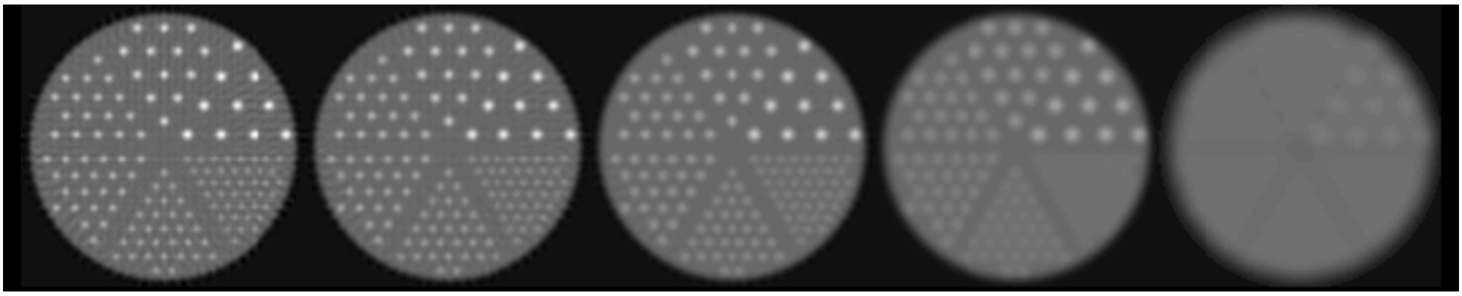
➡ ensures the highest spatial resolution at the expense of noise



- Hann filter

$$w(\rho) = 0.5.(1+\cos\pi\rho/\rho_c) \quad \text{si } \rho < \rho_c$$
$$= 0 \quad \text{si } \rho \geq \rho_c$$

➡ affects intermediate frequencies



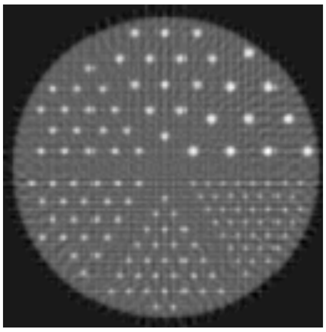
0.5 0.4 0.3 0.2 0.1

Cut-off frequency ρ_c

➡ the lower the cut-off frequency,
the lower the high frequency recovery, i.e.,
the smoother the image

Usual filters : Butterworth filter

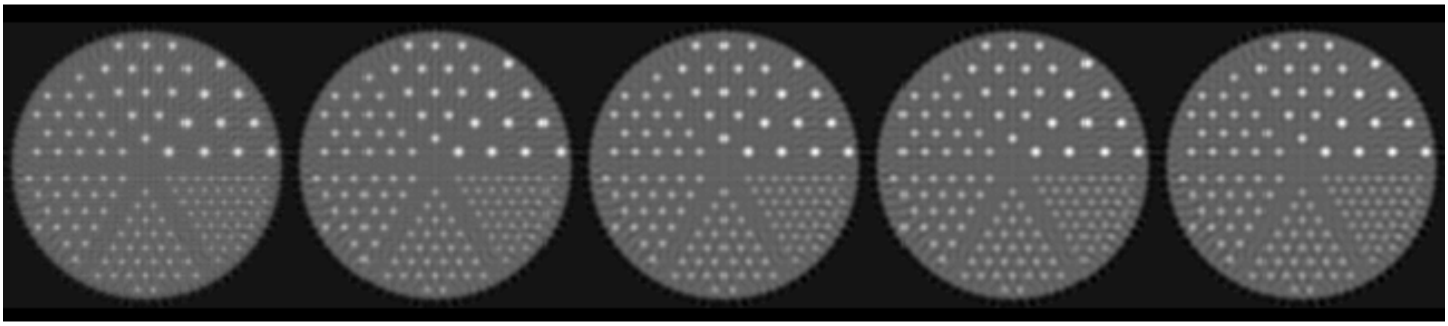
- ramp filter



- Butterworth filter

$$w(\rho) = 1/[1+(\rho/\rho_c)^{2n}] \qquad \text{if } \rho < \rho_c$$

➡ 2 parameters : $\rho_c$cut-off and order n

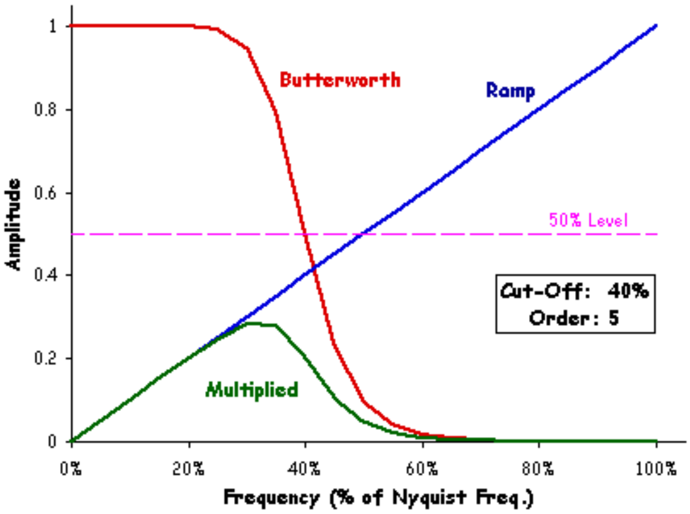


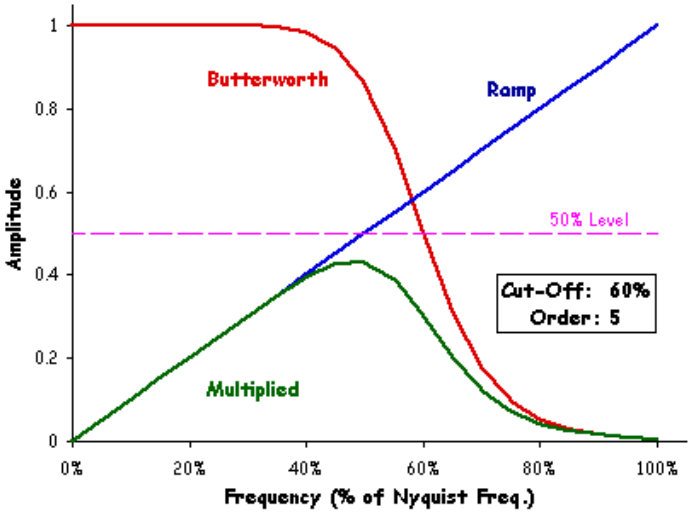
10 8 6 4 2

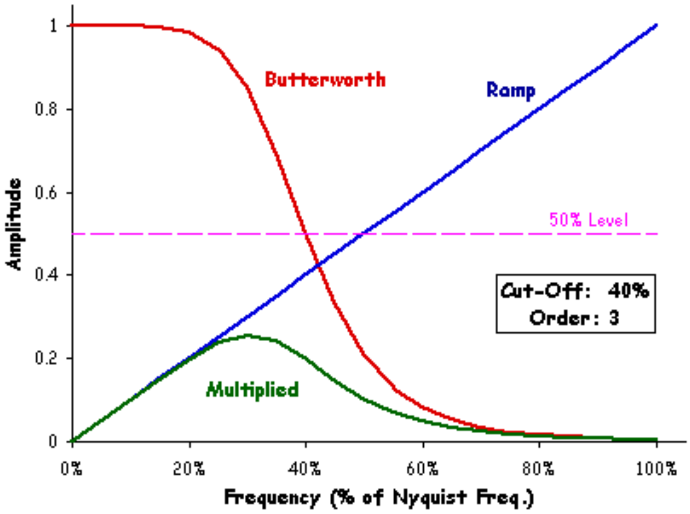
order n, ρ_c=0.25

➡ the higher the order
the lower the high frequency recovery
the smoother the images

Usual filters : Butterworth filter







Filtering: implementation tricks

- Fourier filtering

Convenient property :
A multiplication in the Fourier space is equivalent to a convolution in the spatial domain

$$P(\rho,\theta)\ .\ W'(\rho)$$

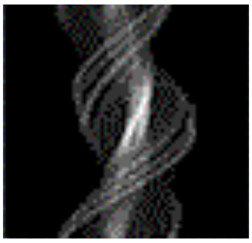
$$\Updownarrow$$

$$p(u,\theta) \otimes w'(u)$$

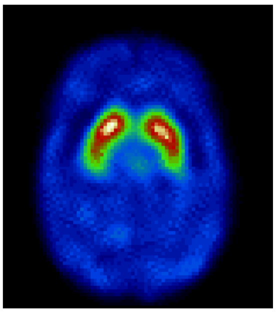
Filtering: several possible implementations

- Fourier filtering

Convenient property :
A multiplication in the Fourier space is equivalent to a convolution in the spatial domain



sinogram



reconstructed slice

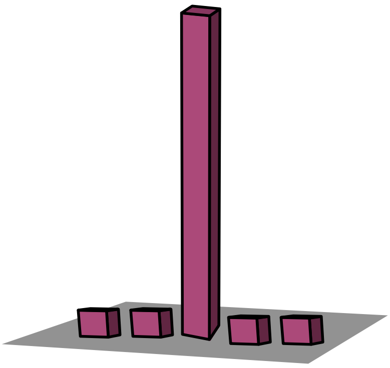
$$\begin{tikzcd}
p(u,\theta) \arrow[d, "FT"] \arrow[drr, "\otimes h(u)"] & & f(x,y) \\
P(\rho,\theta) \arrow[r, "filtering"] & |\rho|\, P(\rho,\theta) \arrow[r, "FT^{-1}"] & p'(u,\theta) \arrow[u, "backprojection"']
\end{tikzcd}$$

- Spatial filtering

Principle of a 1D spatial filtering

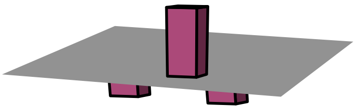
Original projection

1	1	10	1	1



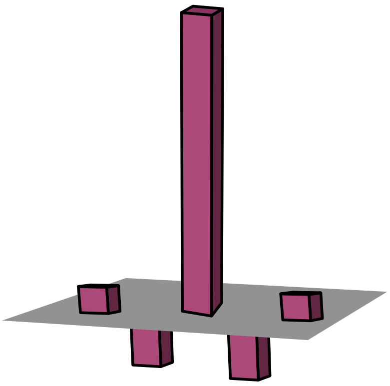
Filter

-0.5	2	-0.5



Filtered projection

1	-3.5	19	-3.5	1



Example of filtering in the projection space

- Calculate the filtered backprojection with the (-0.5 ; 2 ; -0.5) filter of the measured projections (repeat the edge values)

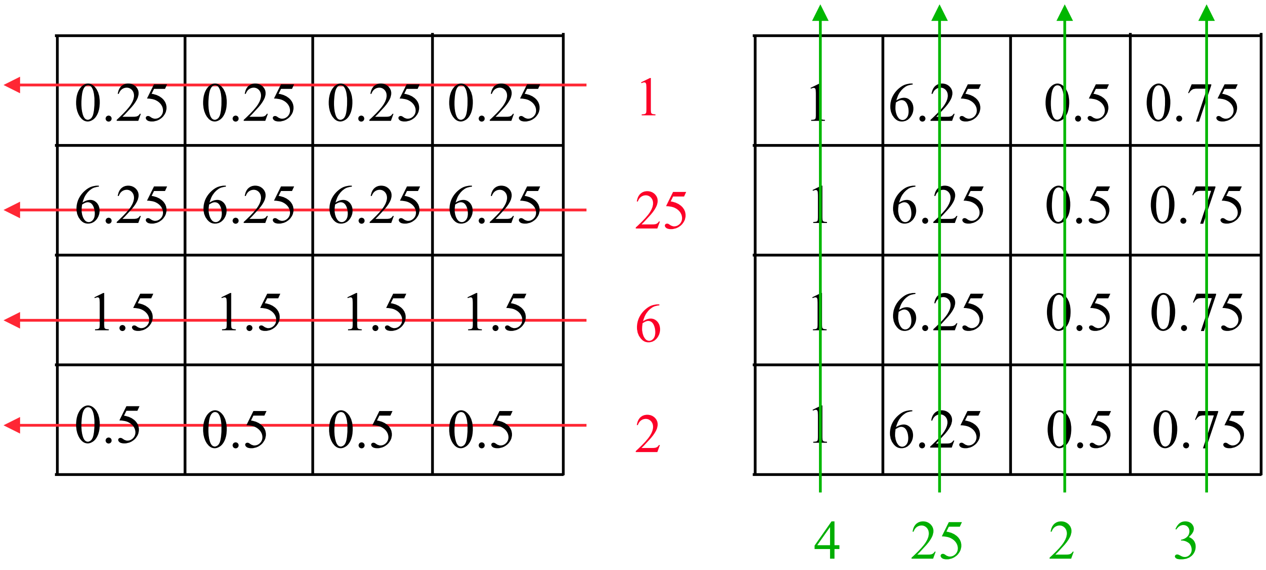
				6
				16
				8
				4
8	16	6	4	

				1
				25
				6
				2
4	25	2	3	

0.25	0.25	0.25	0.25	1
6.25	6.25	6.25	6.25	25
1.5	1.5	1.5	1.5	6
0.5	0.5	0.5	0.5	2

1	6.25	0.5	0.75
1	6.25	0.5	0.75
1	6.25	0.5	0.75
1	6.25	0.5	0.75
4	25	2	3

Example of filtering in the projection space



0.25	0.25	0.25	0.25	1
6.25	6.25	6.25	6.25	25
1.5	1.5	1.5	1.5	6
0.5	0.5	0.5	0.5	2

1	6.25	0.5	0.75
1	6.25	0.5	0.75
1	6.25	0.5	0.75
1	6.25	0.5	0.75
4	25	2	3

Mean :

tumor / bckdg ratio
= 6.25 / 1.4 = 4.5

0.62	3.25	0.4	0.5
3.6	6.25	3.37	3.5
1.25	3.87	1	1.1
0.75	3.37	0.5	0.62

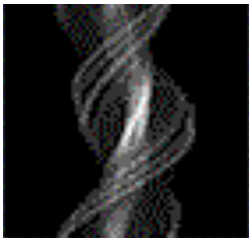
Original image :

tumor / bckdg ratio
= 10 / 1 = 10

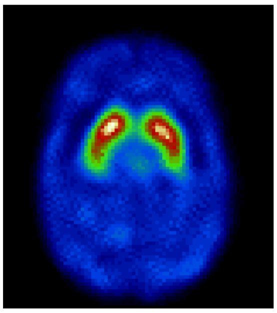
2	2	2	0
2	10	2	2
3	2	2	1
1	2	0	1

2D spatial filter

- Filtering the reconstructed images



sinogram

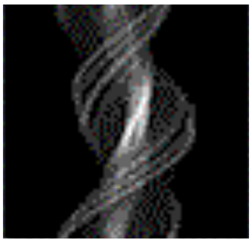


reconstructed slice

$$\begin{tikzcd}
p(u,\theta) \arrow[d, "FT"] & & f(x,y) \\
P(\rho,\theta) \arrow[r, "filtering"] & |\rho|\, P(\rho,\theta) \arrow[r, "FT^{-1}"] & p'(u,\theta) \arrow[u]
\end{tikzcd}$$

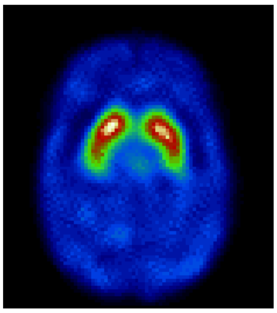
2D spatial filter

- Filtering the reconstructed images



sinogram

$p(u,\theta)$



reconstructed slice

$f(x,y)$

spatial filtering

$\otimes\ g(x,y)$

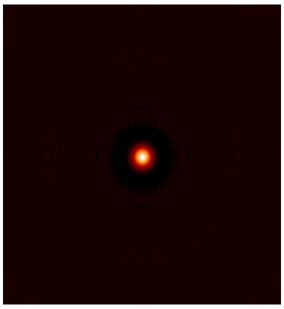
backprojection

$f'(x,y)$

Principle of a 2D spatial filte

Original image

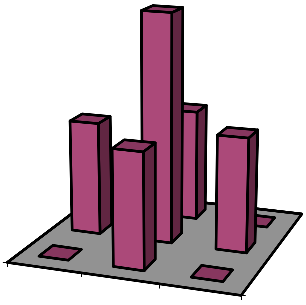
0	0	0	0	0
0	0	0	0	0
0	0	10	0	0
0	0	0	0	0
0	0	0	0	0



Filter

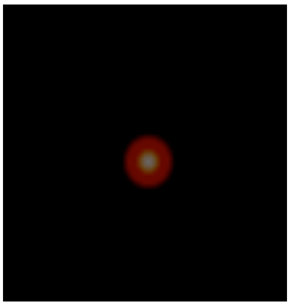
1/6

0	1	0
1	2	1
0	1	0



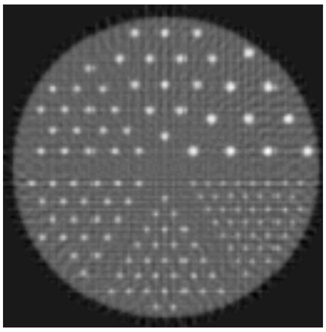
Filtered image

0	0	0	0	0
0	0	1.7	0	0
0	1.7	3.3	1.7	0
0	0	1.7	0	0
0	0	0	0	0



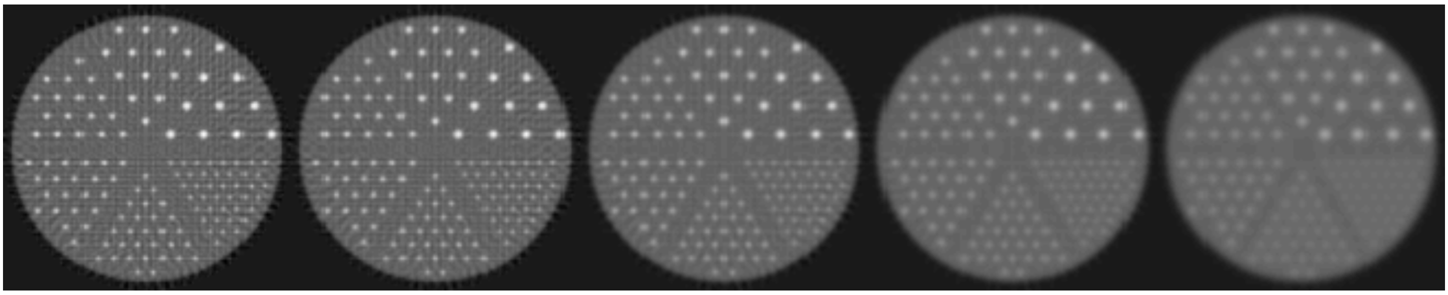
Usual filter : Gaussian filter

- Ramp filter



- Gaussian filter (spatial domain)

$$c(x) = (1/\sigma\sqrt{2\pi}).\exp[-(x-x_0)^2/2\sigma^2]$$



0 1 2 3 4

$$\text{FWHM} = 2\sqrt{2\ln 2}\ \sigma\ \text{(pixel)}$$

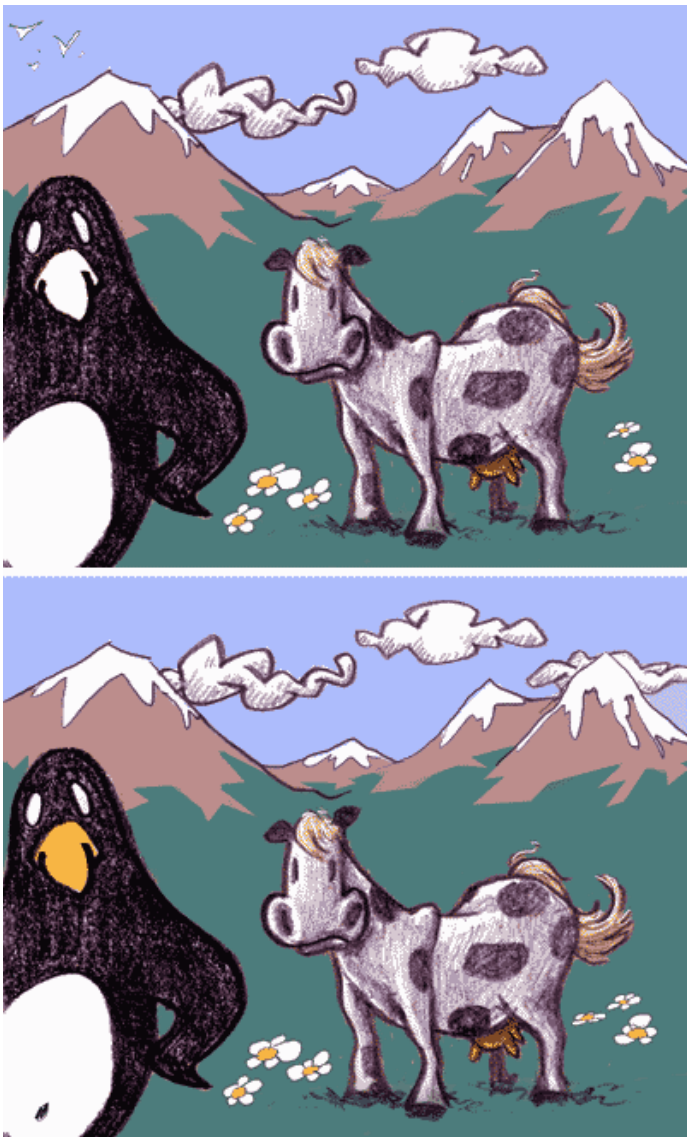
↑

Sets the spatial extent of the filter

➡ the larger the FWHM (or σ),
the smoother the images
the lower the high frequency recovery

Filter implementation: summary

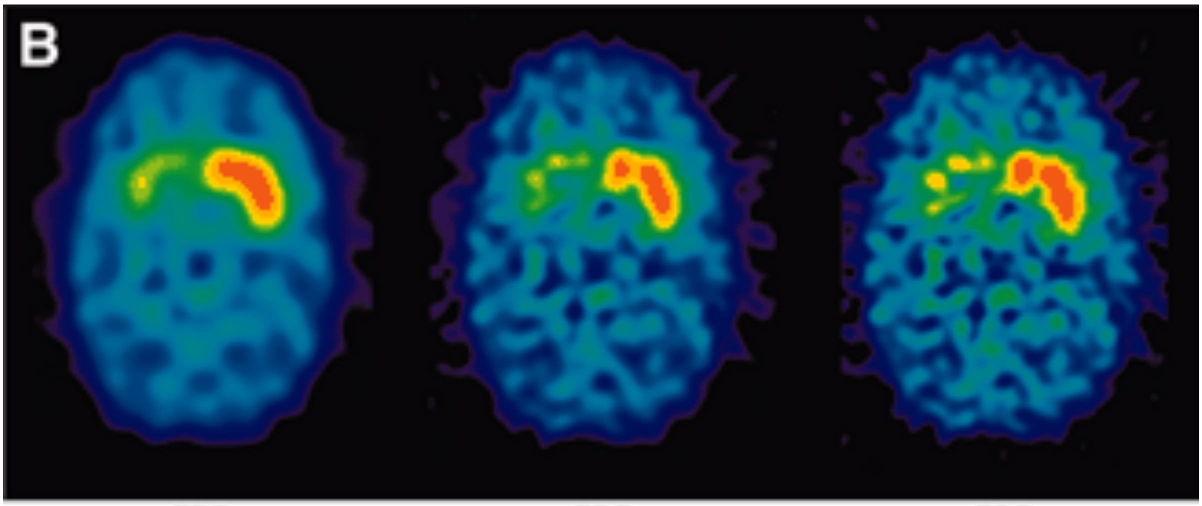
There are several manners to implement a given filter: the same filter implemented differently might yield small differences in the results



beak, penguins, bellybutton, birds, flowers , clouds, snow, horse
ear, mountain.

What is the best filter?

- A given filter is not adapted to all situations



FBP
Butterworth 5th order
0.6 cycles/cm

FBP
Butterworth 5th order
0.9 cycles/cm

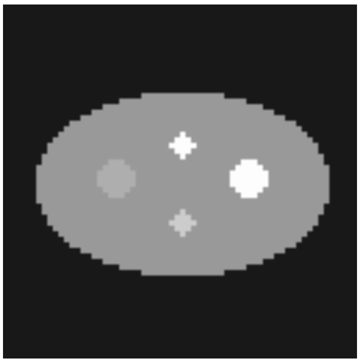
FBP
Butterworth 5th order
1.0 cycles/cm

Koch et al, J Nucl Med 2005

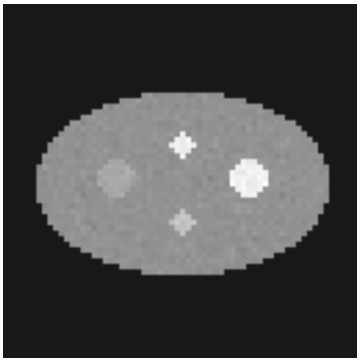
The filter and filter parameters should ideally be optimized as a function of the imaging task (eg, lesion detection, parameter estimate from the image), of the statistics in the raw data, aso

Correlated noise in FBP images

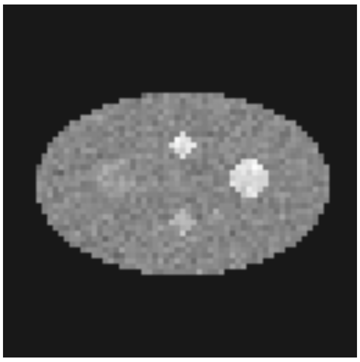
The filtering step introduces noise correlation in the reconstructed images



Original slice
noiseless

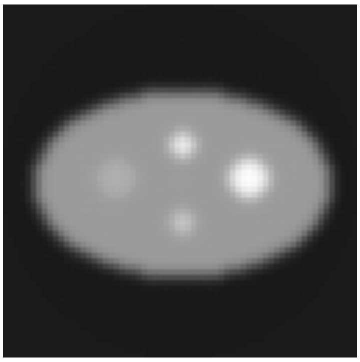


Original noise
with Poisson noise
added
(1 M events)

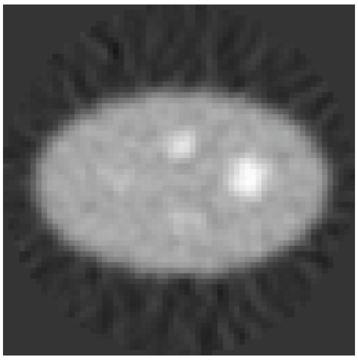


Original noise
with Poisson noise
added
(100 000 events)

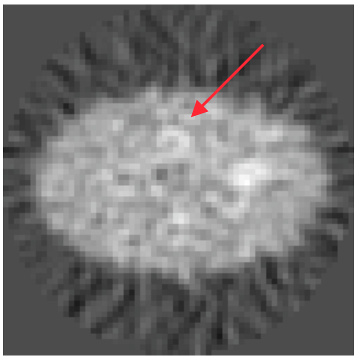
Non spatially correlated noise



Reconstructed slice
FBP



Reconstructed slice
FBP



Reconstructed slice
FBP

Correlated noise

Correlated noise may look like signal !

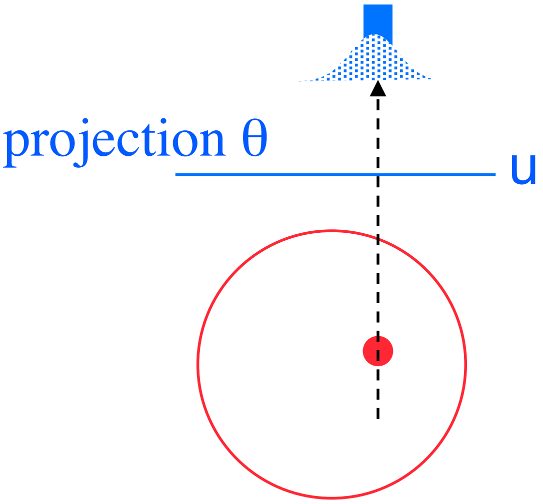
Analytical reconstruction: discussion

- Fast, easy to implement

- Linear (twice the projection values, twice the reconstructed values)

- Spatial resolution / noise trade-off can be tuned using the filter

- Yet, includes many approximations:
    - line integral model (assumes that the detector spatial resolution is ideal, Dirac)



  - no modelling of the noise in the projection data
  - no modelling of the physics (photon attenuation and scattering)
  - data are noisy and sampled, solution is thus neither accurate nor unique

➡ Alternative approach: discrete or iterative reconstruction

Summary (2)

- Analytical reconstruction
  - Principle
  - Central slice theorem
  - Filtered backprojection
  - Filters

• Theoretically exact Radon transform inversion is possible using a Ramp filter. If the data were continuous and noiseless, the filtered backprojection algorithm would then provide the exact solution.

• The ramp filter cannot be used alone on real data, that are always noisy and discrete. An apodization window is used, usually resulting in a low pass filter (Hann, Gaussian), than can be tuned using 1 or 2 parameters and implemented in the spatial or Fourier domain.

• Filtered backprojection remains an approximate solution to tomographic reconstruction, because of a number of underlying assumptions that are not satisfied in real data (noiseless projections, continuous projections, perfect spatial resolution of the detector, no particle matter interactions except when the particle is detected)

Two approaches for tomographic reconstruction

- Analytical methods

$$R[f(x,y)] = \int_0^{\pi} p(u,\theta)\, d\theta$$

- Discrete or iterative methods

$$p = R\, f$$

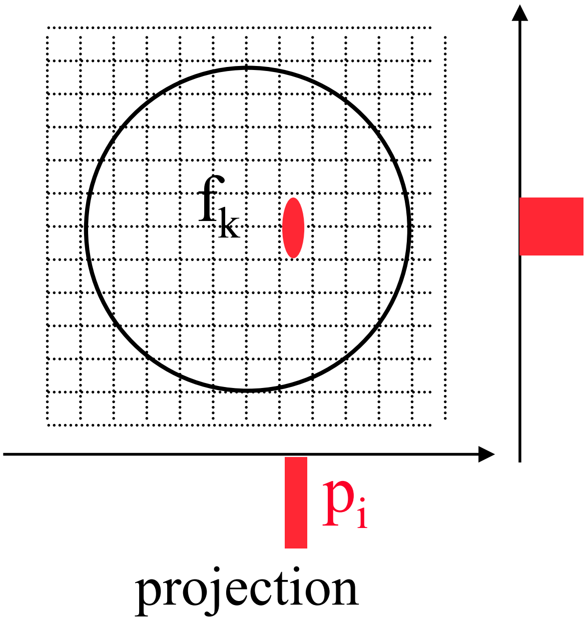
Iterative reconstruction: introduction

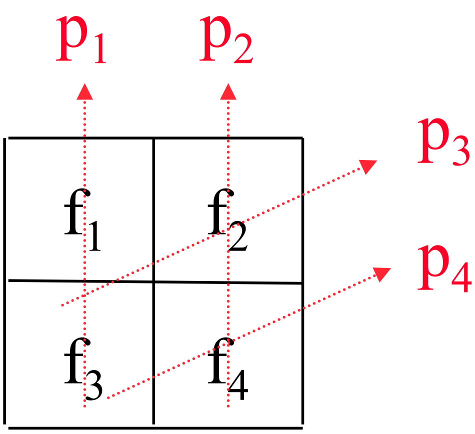
- Discrete expression of the problem using matrix and vectors

$$\begin{bmatrix}p_1\\p_2\\p_3\\p_4\end{bmatrix}=\begin{bmatrix}r_{11}&\cdots&r_{14}\\\vdots&\ddots&\vdots\\r_{41}&\cdots&r_{44}\end{bmatrix}\begin{bmatrix}f_1\\f_2\\f_3\\f_4\end{bmatrix}$$

- Inversion of the corresponding system of equations using an iterative approach

Discrete formalism





$$
\begin{aligned}
p_1 &= r_{11}\,f_1 + r_{12}f_2 + r_{13}f_3 + r_{14}\,f_4\\
p_2 &= r_{21}\,f_1 + r_{22}f_2 + r_{23}f_3 + r_{24}\,f_4\\
p_3 &= r_{31}\,f_1 + r_{32}f_2 + r_{33}f_3 + r_{34}\,f_4\\
p_4 &= r_{41}\,f_1 + r_{42}f_2 + r_{43}f_3 + r_{44}\,f_4
\end{aligned}
$$

In the real world:
large system of equations
128 projections 128 x 128

2 097 152 equations with as many unknown values

Matrix expression of the inverse problem

$$
\begin{aligned}
p_1 &= r_{11}\, f_1 + r_{12}\, f_2 + r_{13}\, f_3 + r_{14}\, f_4 \\
p_2 &= r_{21}\, f_1 + r_{22}\, f_2 + r_{23}\, f_3 + r_{24}\, f_4 \\
p_3 &= r_{31}\, f_1 + r_{32}\, f_2 + r_{33}\, f_3 + r_{34}\, f_4 \\
p_4 &= r_{41}\, f_1 + r_{42}\, f_2 + r_{43}\, f_3 + r_{44}\, f_4
\end{aligned}
$$

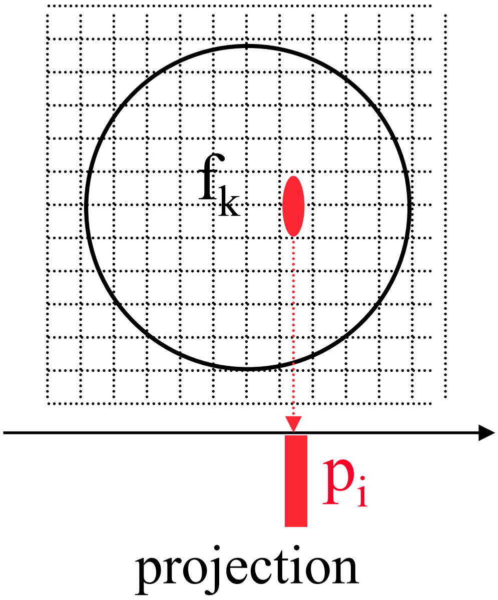
$$
\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}
$$

$$
\underset{\text{projections}}{p} = \underset{\text{projection operator}}{R}\ \underset{\text{object to be reconstructed}}{f}
$$

➡ Problem: find f given p and R

What is R ?

$$p = R\, f$$

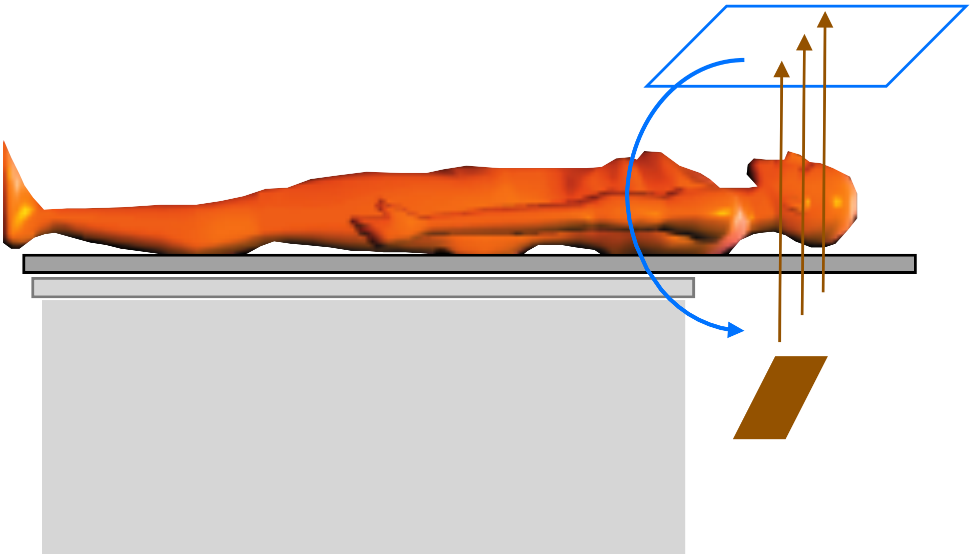


R described the projection process, ie how a signal from the image contributes to a projection measurement:
R models the forward problem

r_{ik} : probability that an « event » emitted in voxel k be detected in pixel i

R = projection operator = system matrix

Dimension of the problem



$$p = R\,f$$

projections   system matrix   object to be reconstructed

$$\begin{aligned}
p_1 &= r_{11}\,f_1 + r_{12}f_2 + \ldots + r_{1F}\,f_F\\
p_2 &= r_{21}\,f_1 + r_{22}f_2 + \ldots + r_{2F}\,f_F\\
&\ldots\\
p_P &= r_{P1}\,f_1 + r_{P2}f_2 + \ldots + r_{PF}\,f_F
\end{aligned}$$

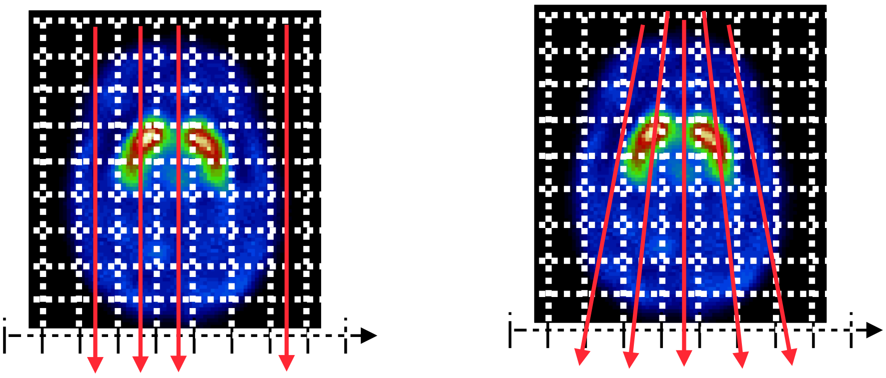
$$\begin{bmatrix} p_1 \\ p_2 \\ \ldots \\ p_P \end{bmatrix} = \begin{bmatrix} r_{11} & \cdots & r_{1F} \\ \vdots & \ddots & \vdots \\ r_{P1} & \cdots & r_{PF} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \ldots \\ f_F \end{bmatrix}$$

- Example : 256 projections of 64 rows (axial direction) and 128 columns (projection element)
  - To reconstruct one slice:
    128 x 256 equations
    128 x 128 unknowns
    R is a (128 x 256 ; 128 x 128) matrix

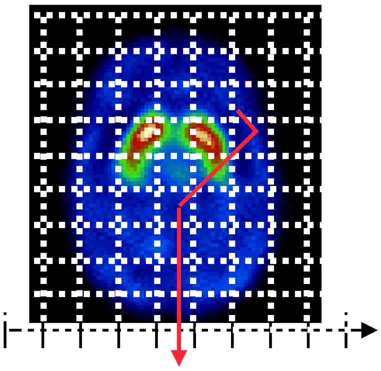
What does R model?

Two aspects

- Modelling of the detection geometry

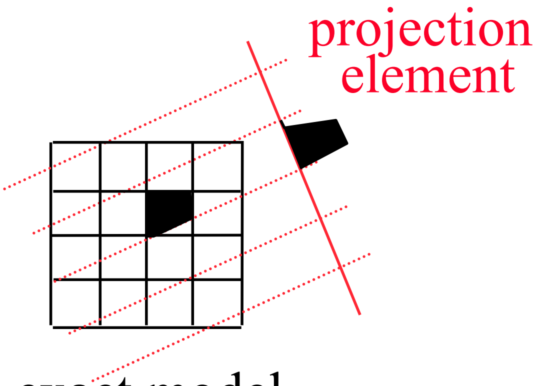


- Modelling of the physics

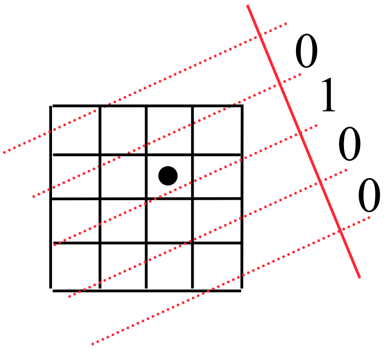


Modelling of the detection geometry

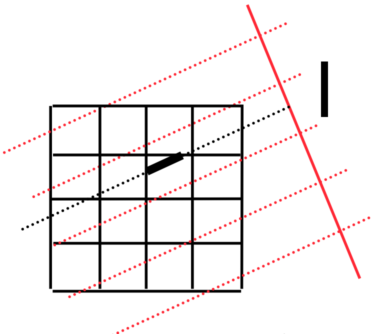
- First: model of the distribution of voxel intensity: describes the contribution of a voxel to a projection bin



exact model
= uniform model



the simplest: Dirac model

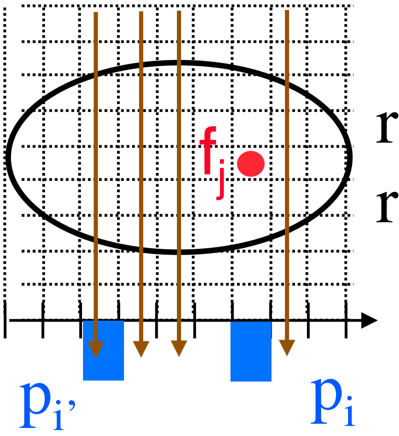


the most convenient
computationally speaking:
line length model

Modelling of the detection geometry

- Second: model of the detector geometry (collimation)

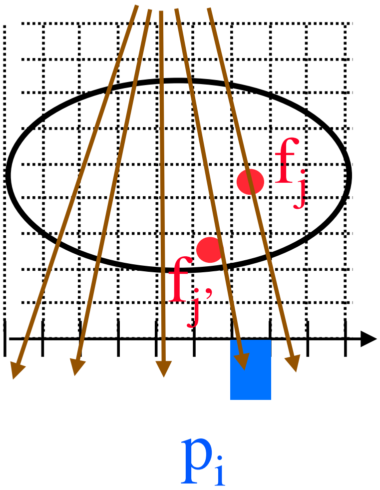
parallel geometry



$r_{ij} = \alpha \neq 0$

$r_{i'j} = 0$

fan beam geometry

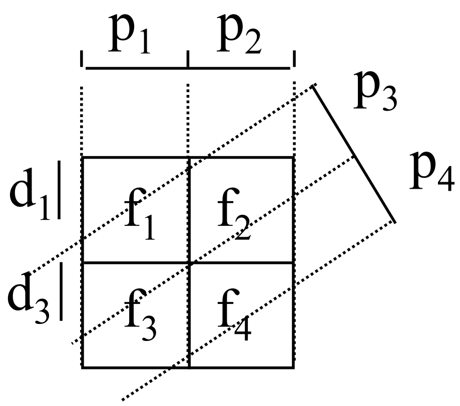


$r_{ij} = 0$

$r_{ij'} = \alpha \neq 0$

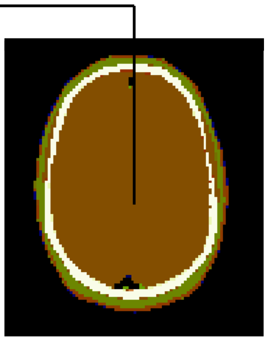
Modelling the physics in R (1)

- Photon attenuation (SPECT and PET)



geometric contribution

$$p_1 = \boxed{g_{11}} f_1 \exp(-\mu_1 d_1)$$
$$+ g_{13} f_3 \ \exp(-\mu_3 d_3 - 2\ \mu_1 d_1)$$



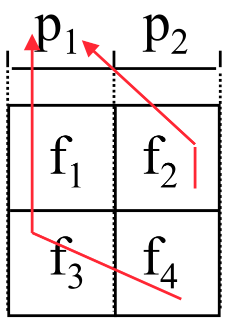
μ map

In that case:

$$r_{11} = g_{11} \exp(-\mu_1 d_1)$$
$$r_{13} = g_{13} \ \exp(-\mu_3 d_3 - 2\ \mu_1 d_1)$$

Modelling the physics in R (2)

- Scattering (SPECT and PET)



without scatter modelling :

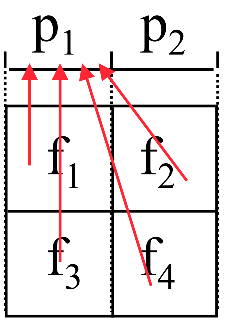
$$p_1 = r_{11}\, f_1 + r_{13}\, f_3$$

with scatter modelling:

$$p_1 = r_{11}\, f_1 + r_{12}\, f_2 + r_{13}\, f_3 + r_{14}\, f_4$$

Modelling the physics in R (3)

- Detector response

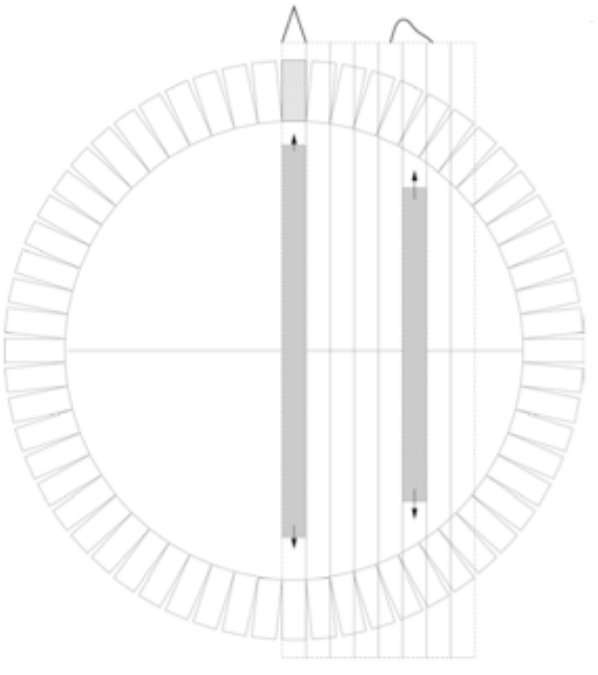


without point spread function (PSF) modelling

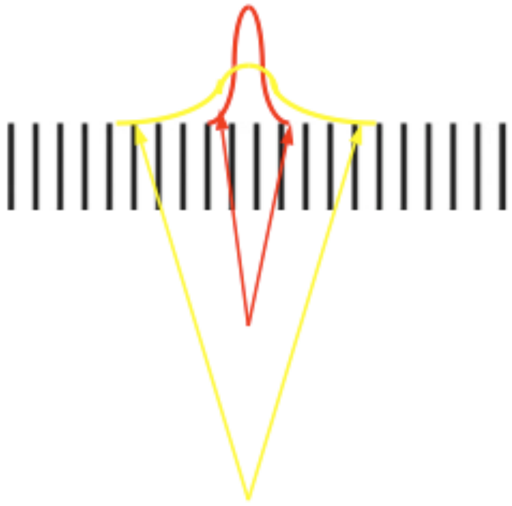
$$p_1 = r_{11} f_1 + r_{13} f_3$$

with PSF modelling :

$$p_1 = r_{11} f_1 + r_{12} f_2 + r_{13} f_3 + r_{14} f_4$$



PET



SPECT

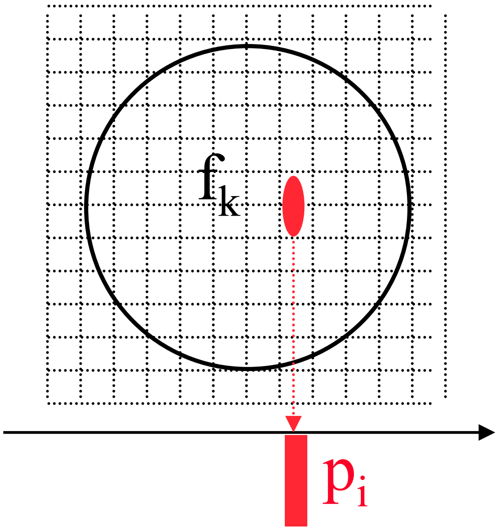
Modelling the physics in R (4)

- Even more advanced R models

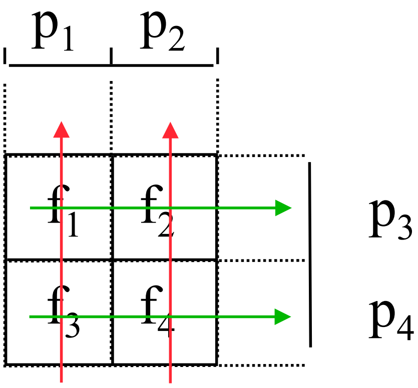
  - modelling of the patient respiratory motion in R (research)
  - modelling the mean free positron path in PET (research)

No theoretical limitations: we could a priori model all phenomena impacting the R element values, that is the probability that a photon emitted in voxel k be detected in projection bin i

R operator



projection



$$p_1 = f_1 + f_3$$
$$p_2 = f_2 + f_4$$
$$p_3 = f_1 + f_2$$
$$p_4 = f_3 + f_4$$

$$\Downarrow$$

$$R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

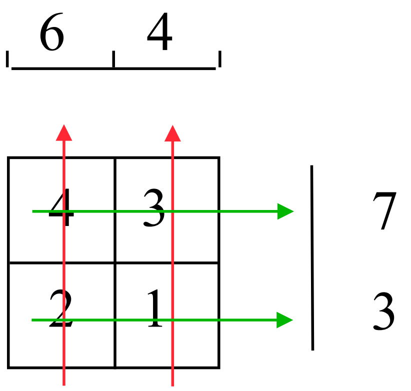
In practice

$$R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

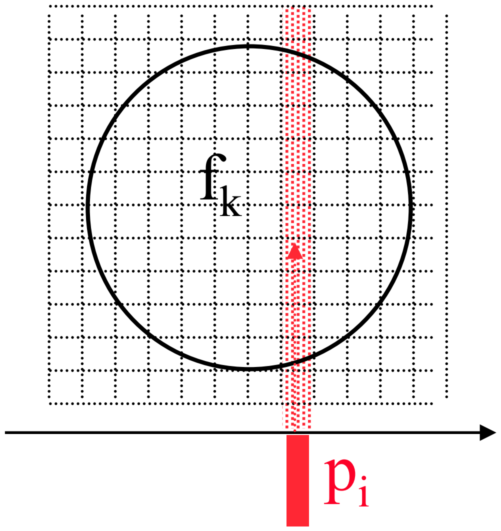
Please calculate the projections of (4 3 2 1) :

$$\begin{array}{c} 6 \\ 4 \\ 7 \\ 3 \end{array} \begin{bmatrix} p_1 \\ p_2 \\ \dots \\ p_P \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

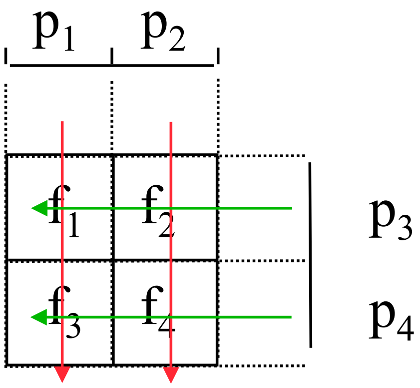
This exactly corresponds to:



Backprojection operator



backprojection



$$f^*_1 = p_1 + p_3$$
$$f^*_2 = p_2 + p_3$$
$$f^*_3 = p_1 + p_4$$
$$f^*_4 = p_2 + p_4$$

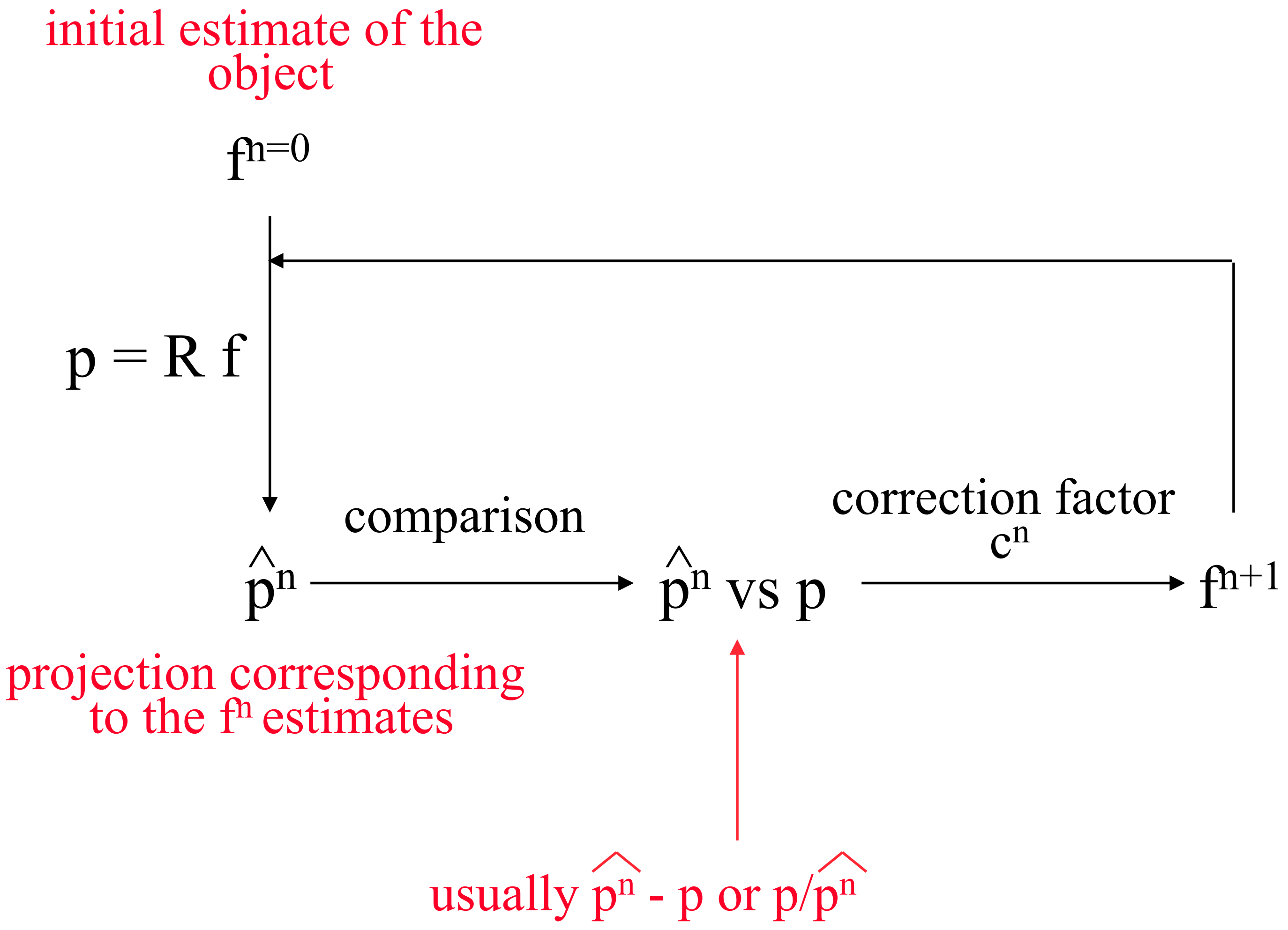
$$\Downarrow$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} = R^t$$

Solution of the inverse problem

$$p = R\,f$$

Finding a solution f mimizing a distance d(p,Rf), p and R being known



initial estimate of the object

$f^{n=0}$

$p = R\,f$

$\hat{p}^n$ comparison $\hat{p}^n$ vs p correction factor c^n f^{n+1}

projection corresponding to the f^n estimates

usually $\hat{p}^n$ - p or p/$\hat{p}^n$

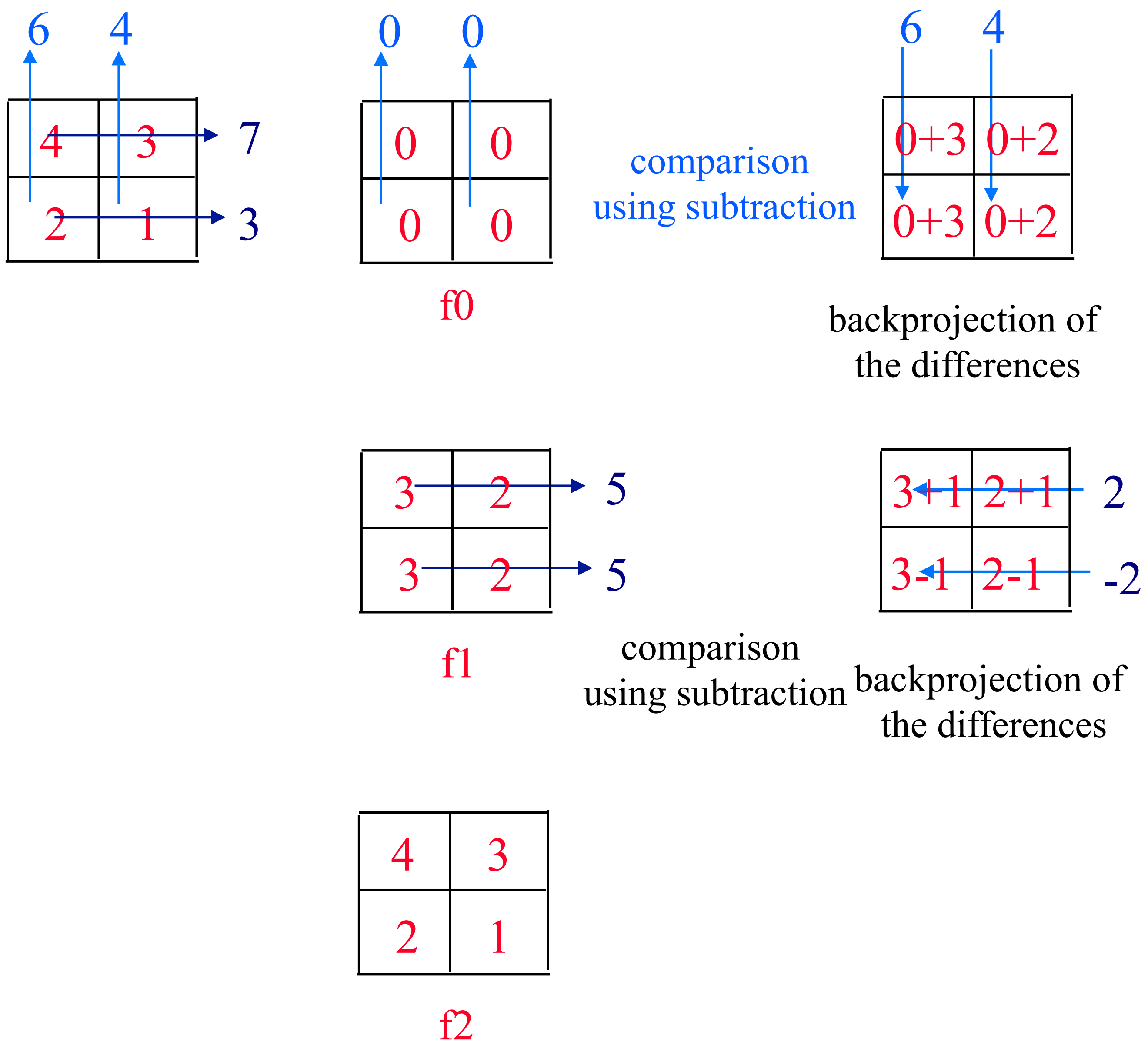
Two classes of iterative methods

- Algebraic methods
  - conventional iterative methods used to solve a linear equation system
  - minimization of $||p - R f||^2$
  - ART, SIRT, ILST, conjugate gradient, etc

- Statistical methods
  - Bayesian estimate
  - account for the noise in the data (Poisson, Gaussian)
  - maximize a likelihood function
  - MLEM, OSEM, RAMLA, DRAMA

Example of algebraic method: ART

- Algebraic reconstruction technique



f0

comparison
using subtraction

backprojection of
the differences

f1

comparison
using subtraction

backprojection of
the differences

f2

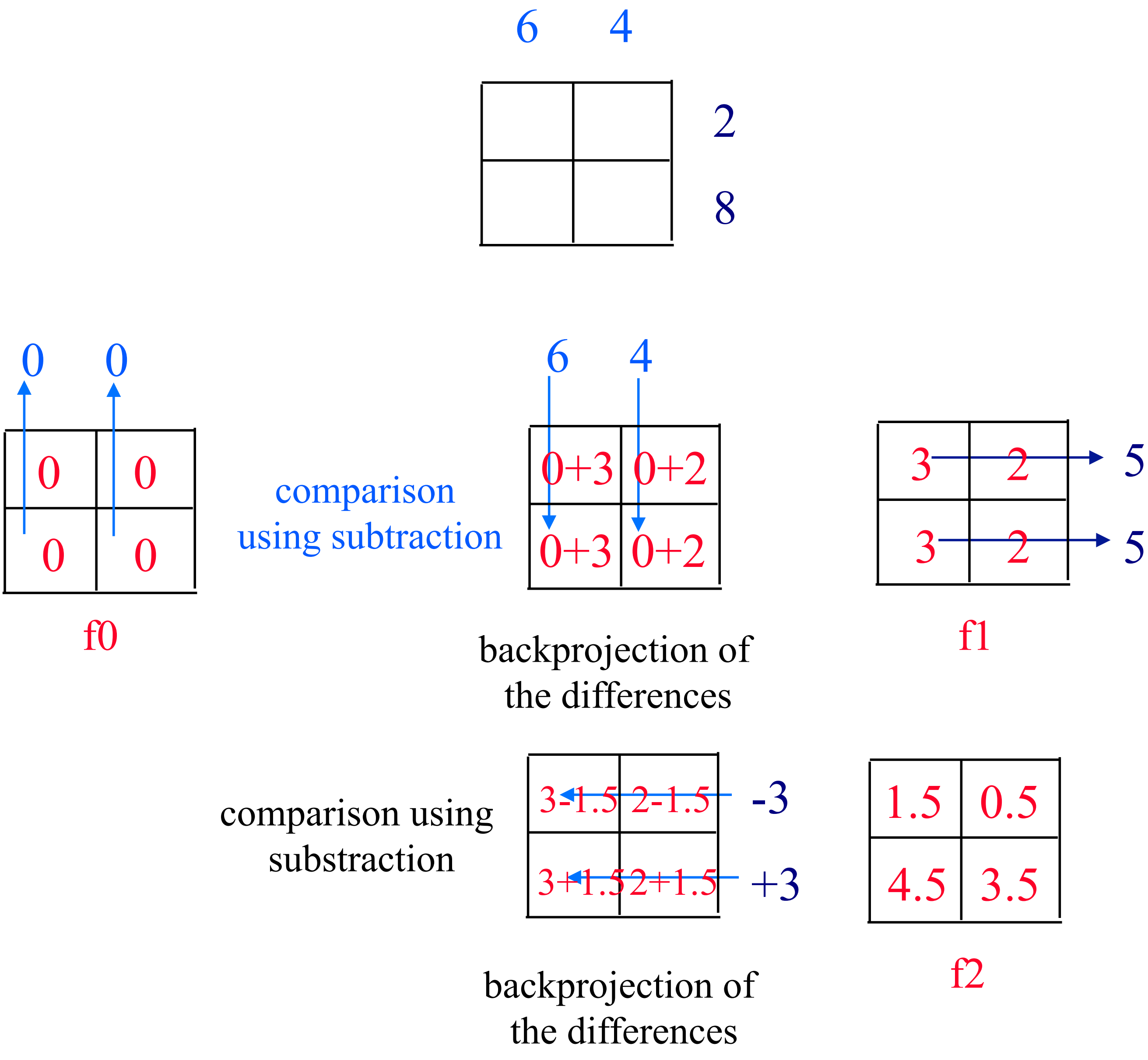
Let’s try!

6	4	
		2
		8

0	0
0	0

f0

Solution using ART



comparison
using subtraction

backprojection of
the differences

comparison using
substraction

backprojection of
the differences

Limitations of algrebraic methods

- They do not account for the noise present in the projections
- They do not include any prior on the solution

Statistical methods offer an appealing alternative as
they can model the statistical properties of:
- the measured projections
- the object to be reconstructed

Why is it so important to model noise?

- in SPECT, PET and CT, Poisson noise (counting)

- PET Gemini TF

\- 44 rings of 644 crystals LSO (4 mm x 4 mm x 22 mm)
\- ~ 4 E8 lines of response defined by 2 crystals

Injection of ~ 10 mCi = 370 MBq
5 min acquisition

Number of β+ desintegrations = 370 E6 x 5 x 60 = 1.11 E11

Attenuation effect: exp(-0.097 x 30) = 0.0544
is 6 E9 coincidences arriving on the detector

Detector efficiency (2%)
is 1.2 E8 detected coincidences

Ie 1.2 E8/ 4 E8 = 0.3 coincidence per LOR !

Most used statistical method: MLEM

- MLEM = Maximum Likelihood Expectation Maximization
- Assumes that the measured data follow a Poisson statistics

1781-1840

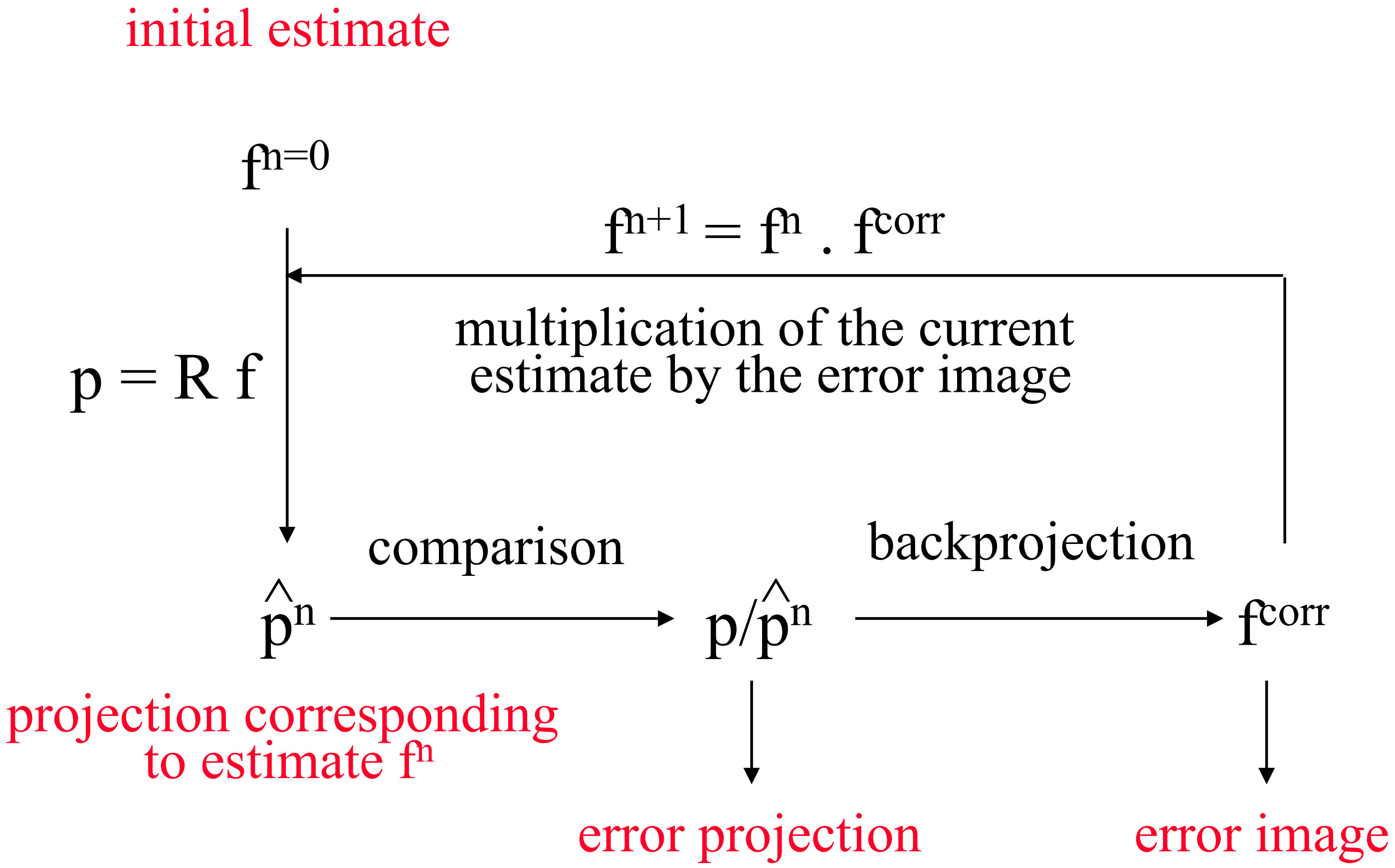
Consistent with the properties of SPECT and PET data

This means that if projections are pre-processed before reconstruction, MLEM assumption is no longer valid !

MLEM algorithm

- Update formula (demonstration is lengthy):

$$f^{n+1} = f^n . R^t [p / p^n]$$



initial estimate

$f^{n=0}$

$f^{n+1} = f^n . f^{corr}$

multiplication of the current
estimate by the error image

$p = R f$

comparison

backprojection

$\hat{p}^n$

$p/\hat{p}^n$

f^{corr}

projection corresponding
to estimate f^n

error projection

error image

MLEM algorithm

Properties:

* solution is always positive or zero
* slow convergence (>100 iterations required)
* iterative images widely used in the clinics (in its accelerated OSEM version)
* NON linear!

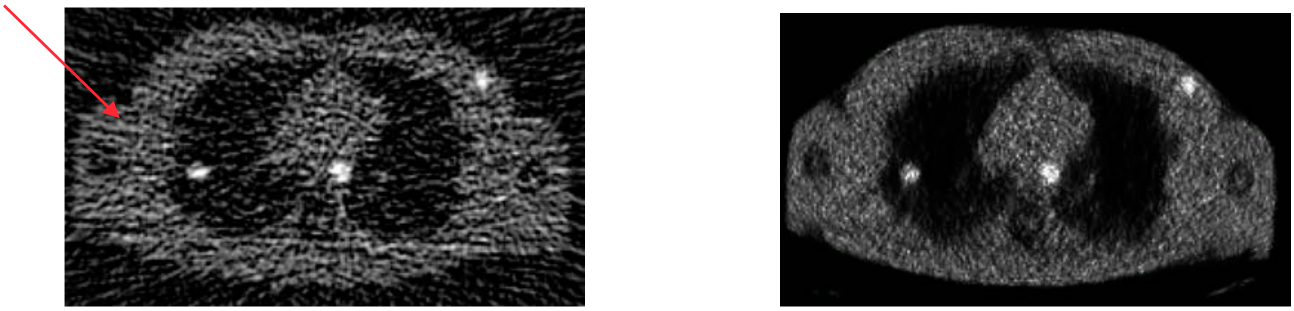
Non linearity is counter intuitive.

* bias (over estimation of low values) in regions with low signal (due to the non negativity constraints inherent to MLEM)

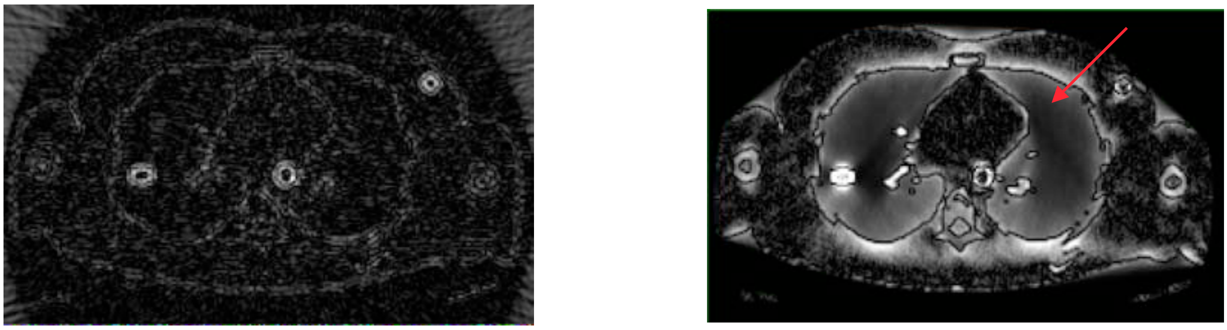
Example

FBP (Hamming)

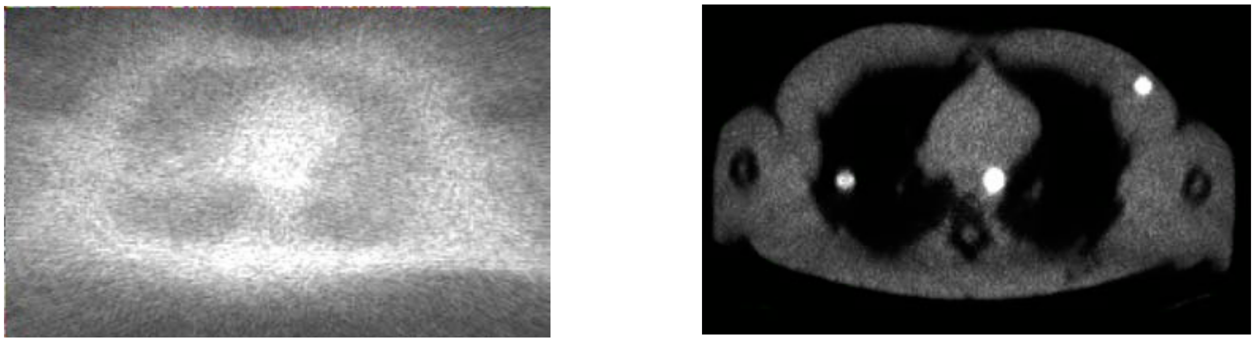
MLEM
(32 itérations)



Reconstructed images



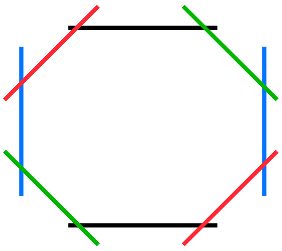
Bias $(\hat{f}\text{-}f)$



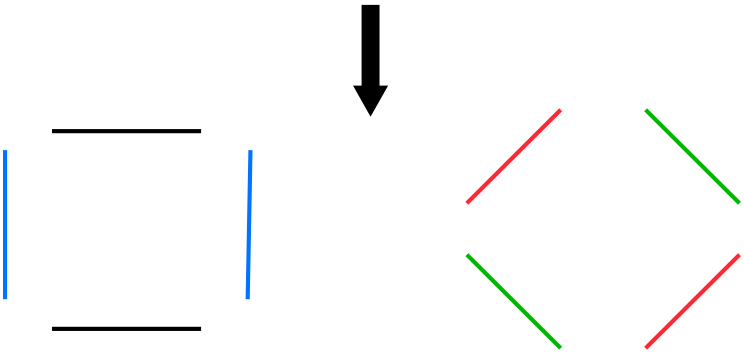
Variance

Acceleration of MLEM : OSEM

- OSEM = Ordered Subset Expectation Maximisation
- Sorting the P projections in ordered subsets
  Exemple :

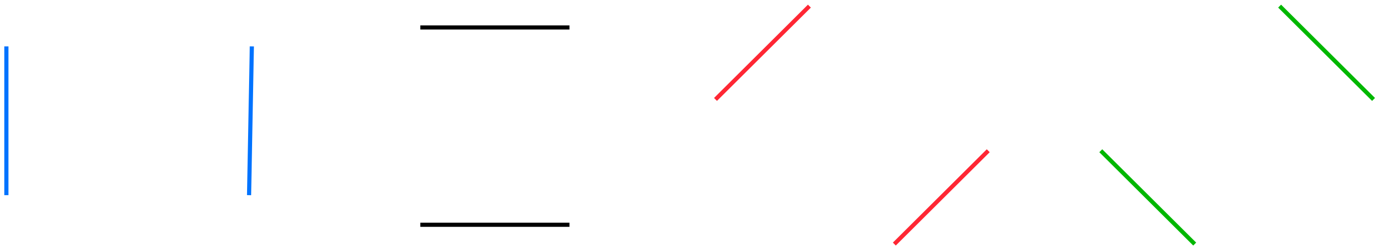


8 projections



2 subsets of 4 projections

or



4 subsets of 2 projections

OSEM

- Using MLEM on the subsets (example of 2 subsets):

  - iteration 1 :

    estimation of f^1 from f^0 and projections p^1
    corresponding to subset 1

    $$f^1 = f^0 . R^t [p / p^1]$$

    estimation of f'^1 from f^1 et projections p'^1
    corresponding to subset 2

    $$f'^1 = f^1 . R^t [p / p'^1]$$

  - iteration 2 :

    estimation of f^2 from f'^1 and projections p^2
    corresponding to subset 1

    $$f^2 = f'^1 . R^t [p / p^2]$$

    estimation of f'^2 from f^2 and projections p'^2
    corresponding to subset 2

    $$f'^2 = f^2 . R^t [p / p'^2]$$

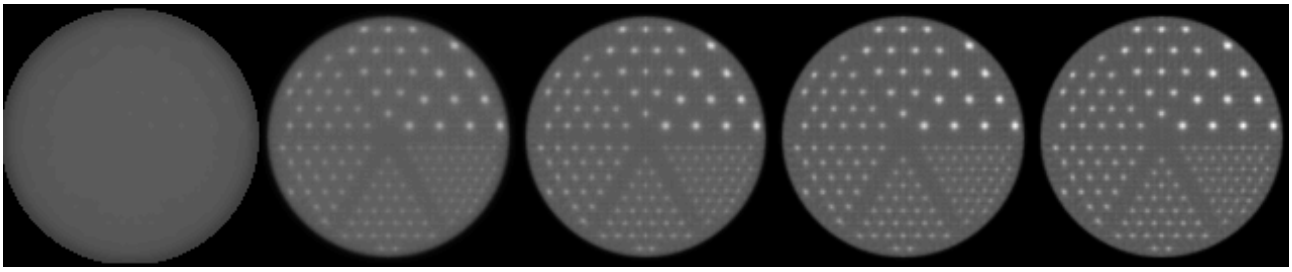
  etc.

OSEM using S subsets and I iterations
⇔ SI iterations of MLEM
but S times faster !!!

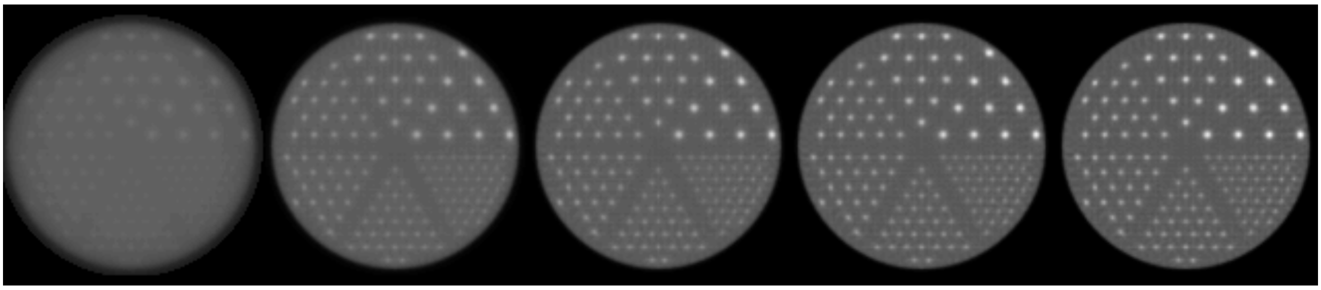
Beware: use at least 4 projections per subset!

Example of OSEM results

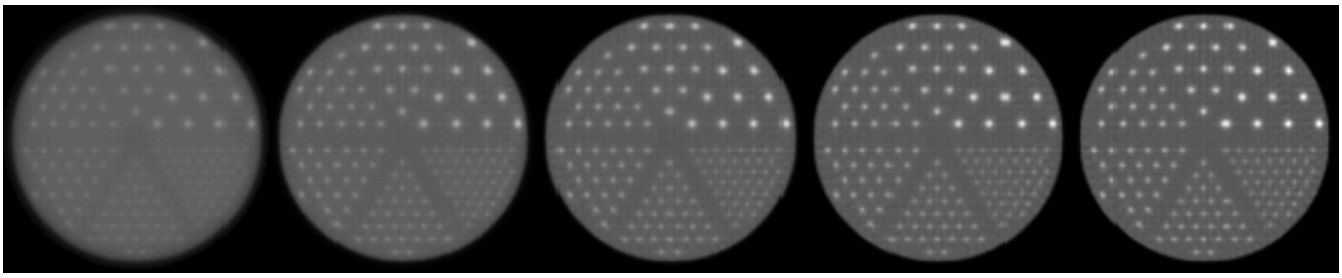
MLEM 1 16 24 32 40 iter.



OSEM 1 4 6 8 10 iter.
4 subsets



OSEM 1 2 3 4 5
8 subsets



OSEM has to be described using a number of subsets
and a number of iterations

Components of an iterative reconstruction algorithm

5 components to be defined:

- description of the f signal representation model
  - usually a matrix of voxels
  - but can be overlapping functions like « blobs »
- system matrix R
  - describes the forward model
  - models the geometry and the physics of the acquisition
- data model for p
  - statistical properties of the data (Poisson, Gauss)
- objective function to be optimized to solve p=Rf for f
  - maximum likelihood
  - maximum a posteriori
  - weighted least squares
  - …
- optimization strategy to optimize the objective function
  - expectation maximization
  - descent algorithm
  - …

Each iterative algorithm can be described using these 5 components and varies as a function of these choices

Many iterative algorithms have been proposed

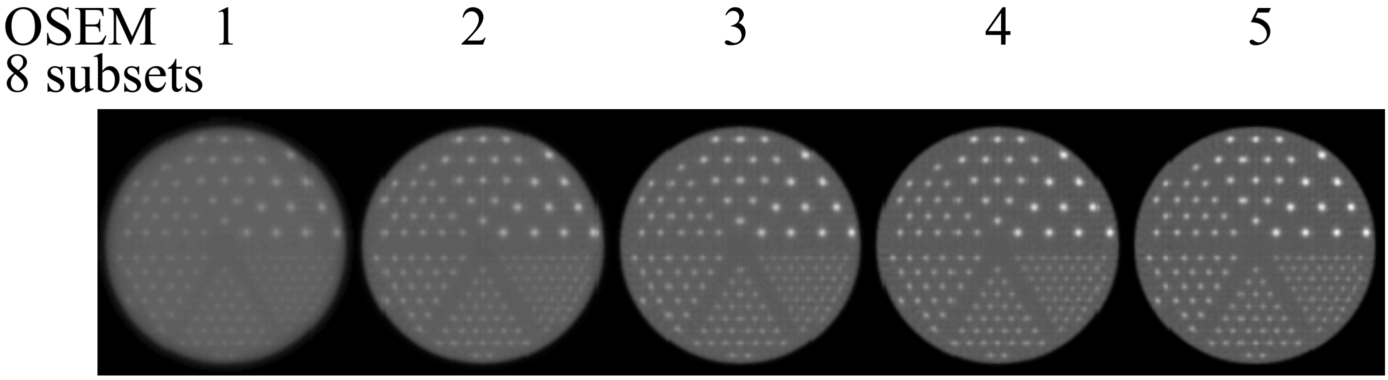
- RAMLA (row action maximum likelihood algorithm) is a OSEM-type algorithm with:
  - a number of subsets equal to the number of projections
  + a relaxation parameter to control noise

- DRAMA, SAGE, SMART, Conjugate gradients, …

- Voxel grid is mostly used for f description, but Philips also used blobs (3D Gaussian functions)

Properties of iterative methods

- The higher the number of iterations, the better the high frequencies recovery



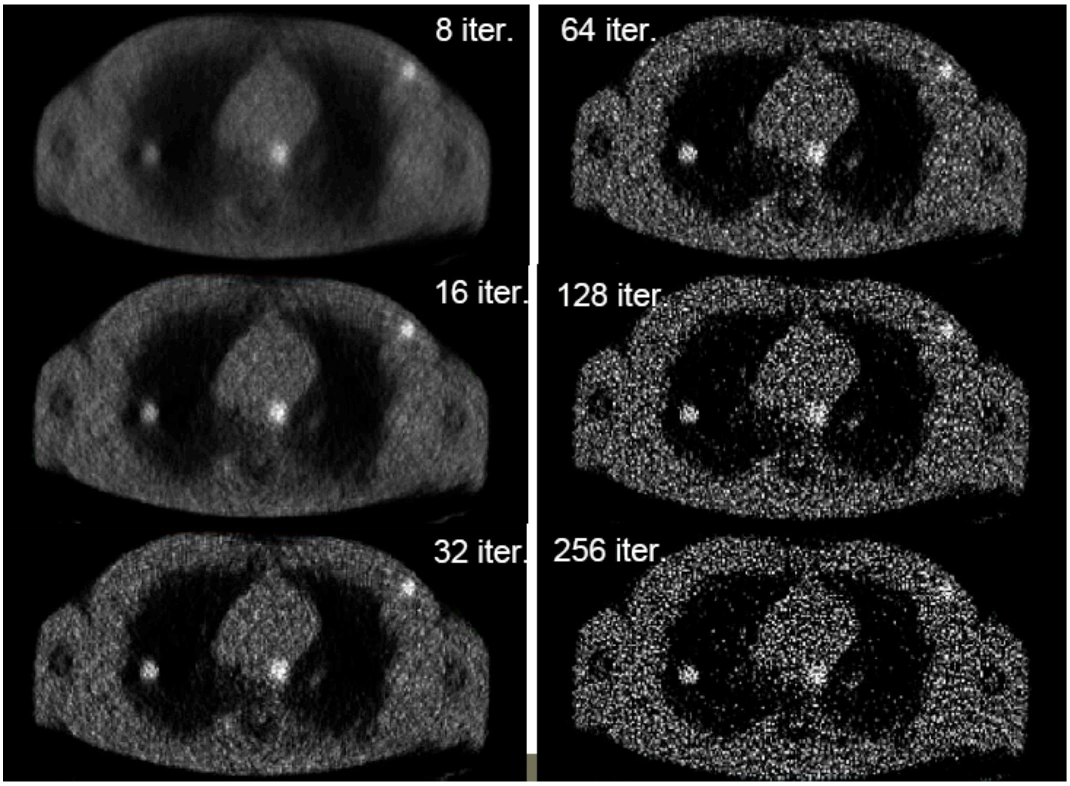
- The number of iterations sets the trade-off between spatial resolution and noise (similar to the filter in FBP)

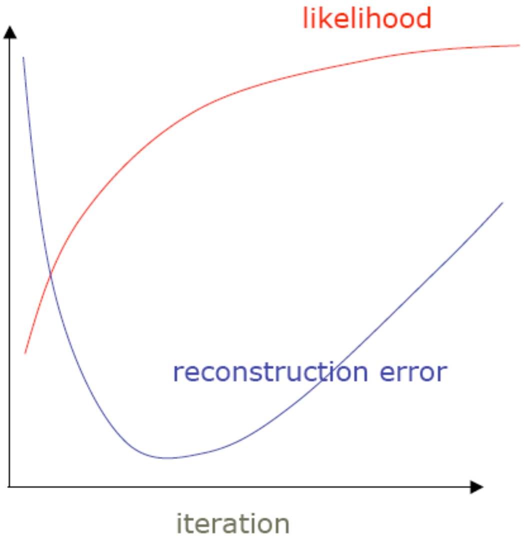
- The number of iterations should always be sufficient to converge, and then regularization should be applied to reduce noise (see later)

Properties of iterative methods

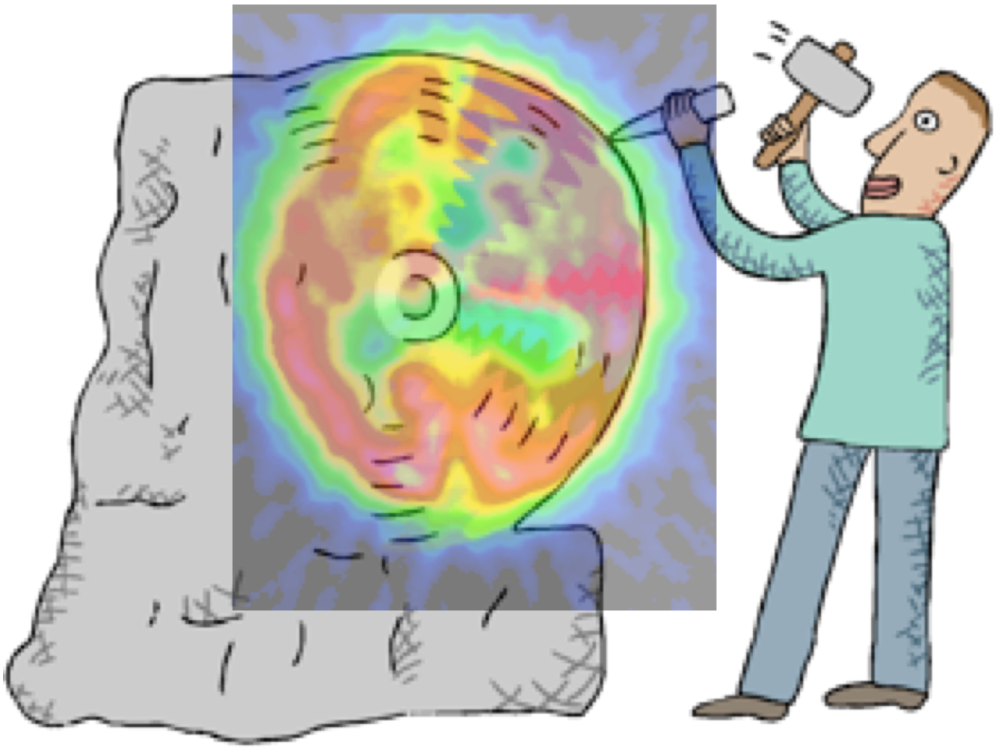
- How to choose the number of iterations?

  - convergence towards the solution followed by divergence to the amplification of noise



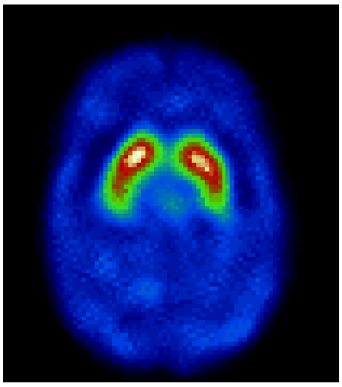


Regularization



Makes the solution close to what is expected

unlikely



likely

Penalizes unlikely solution and favors likely ones using priors

Three approaches for regularization

• Empirical methods:
- post-filtering
- early stop of iterations
- filtering between iterations
- …

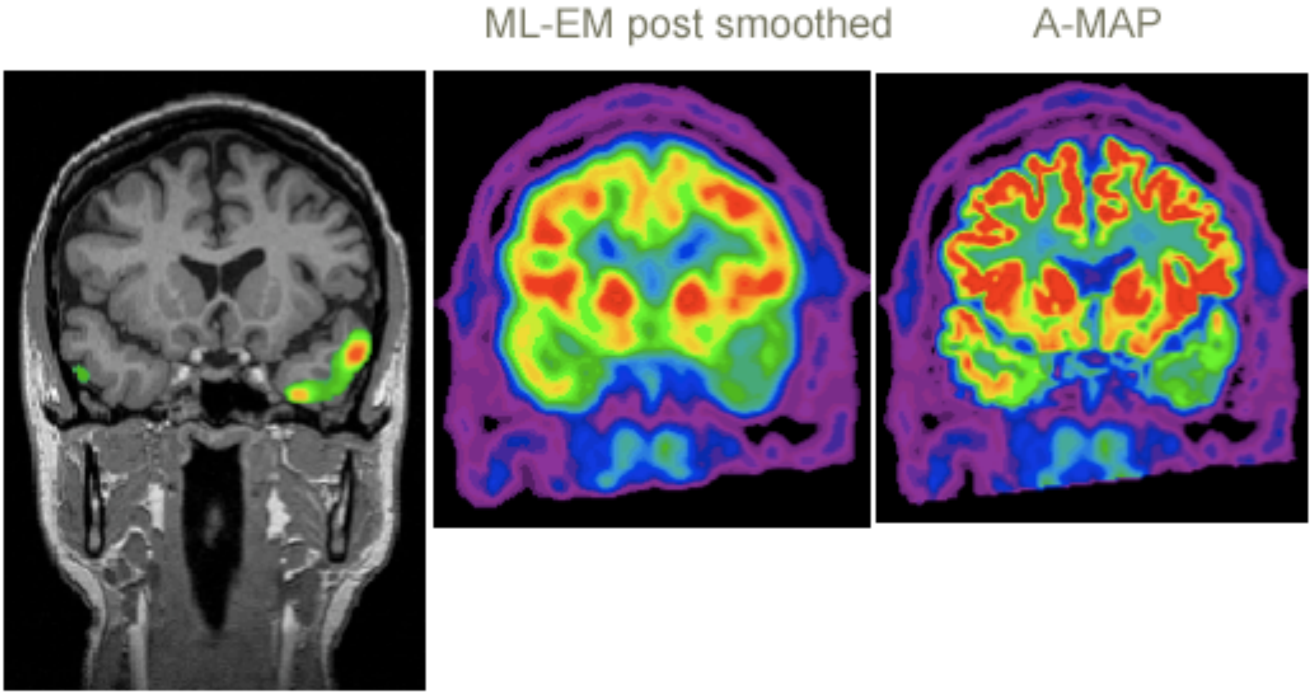
• Variational regularization:
  - solution without regularization:
    minimisation of d(p,Rf)

  - regularized solution:
    minimisation of $d_1(p,Rf) + \lambda d_2(f,f_{apriori})$

• Reduce the dimension of the problem, ie the number of unknown to be estimated
- using blob functions
- using time-dependent basis functions in dynamic imaging …

Example

- Introduction of priors derived from a CT or an MR



ML-EM post smoothed

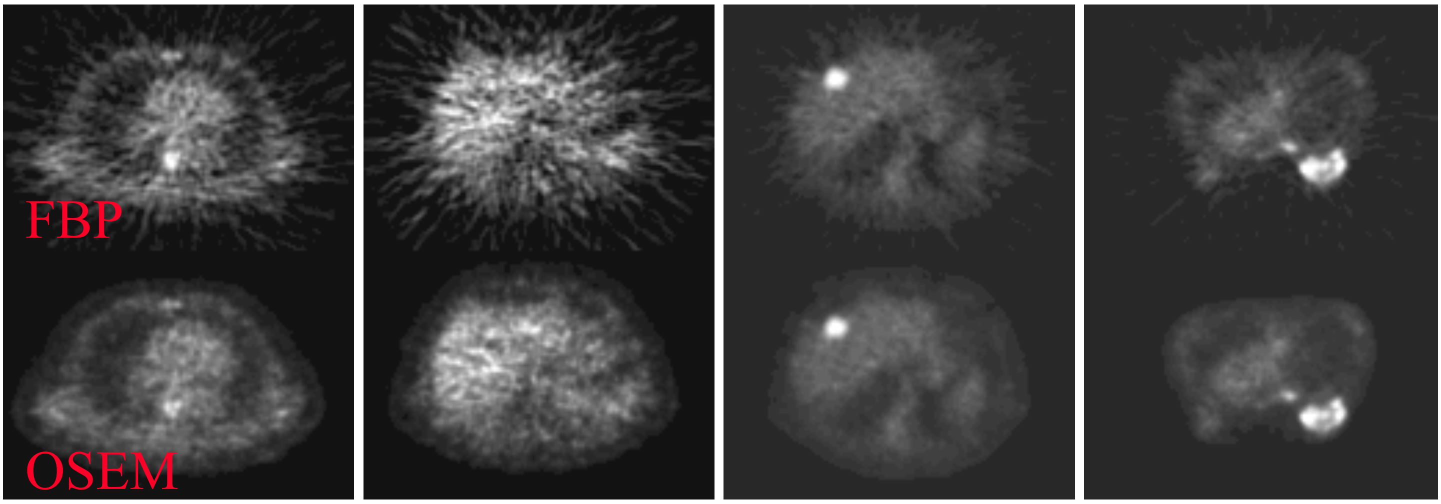
A-MAP

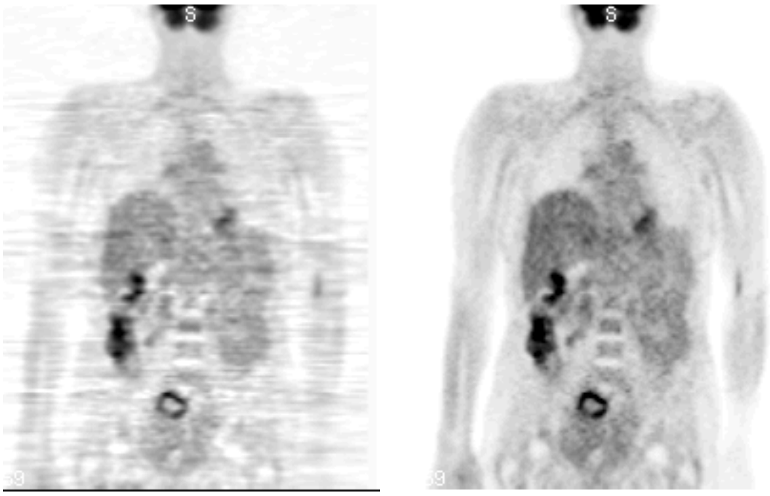
Baete et al, IEEE Trans Med Imaging 2004

Regularization can yield great visual results, but the parameters they require are difficult to adjust automatically

Analytical or iterative reconstruction (1)

PET





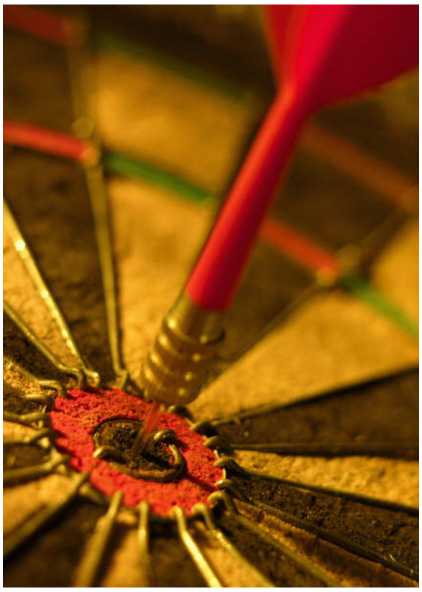
Analytical or iterative reconstruction (2)

- Iterative algorithm with respect to FBP
  - * no streak artefacts
  - * possible modelling of the physics in R
  - * easy management of complicated geometry for which no FBP variant exists
  - * possible modelling of the statistical properties of the measured data
  - * possible introduction of priors

  - * longer computation time
  - * non linear for some of them
  - * some other artefacts (noise correlation)

Analytical or iterative reconstruction (3)

- Current trends towards iterative algorithms:
  - * because modelling the physics is extremely appealing
  - * because of the flexibility in what can be modelled within R (complicated detector geometry)
  - * GPU implementation makes iterative reconstruction fast

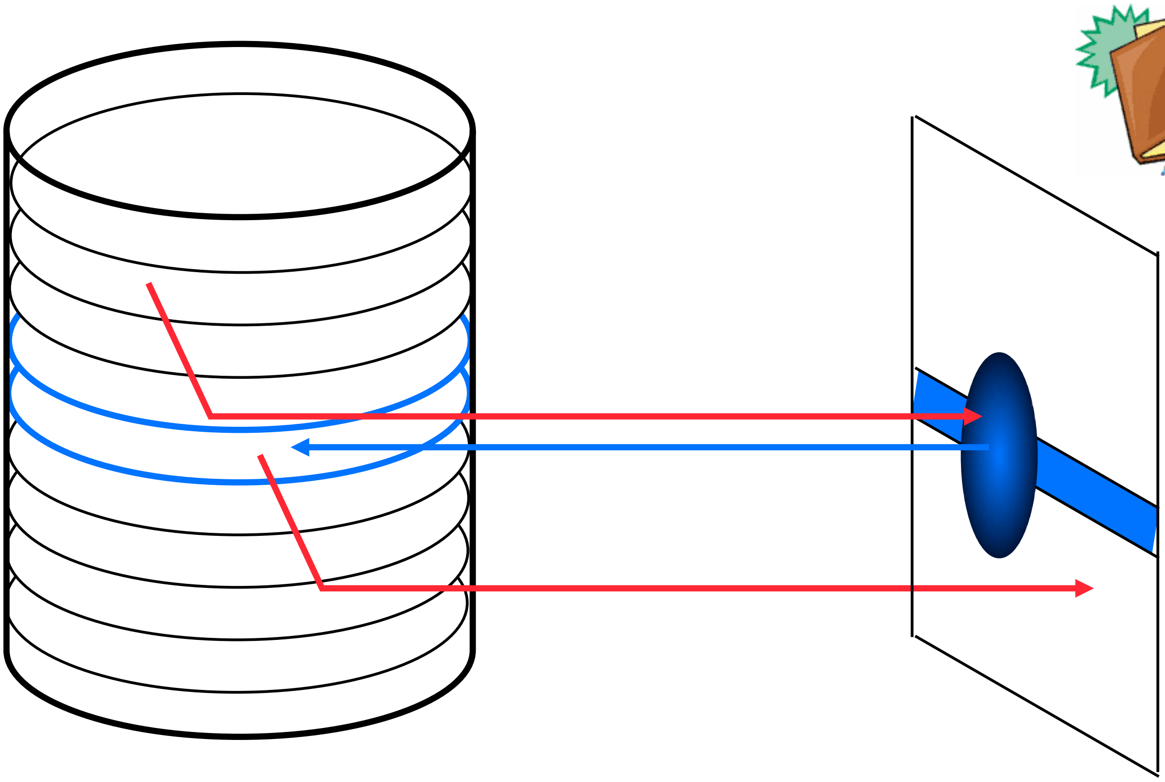


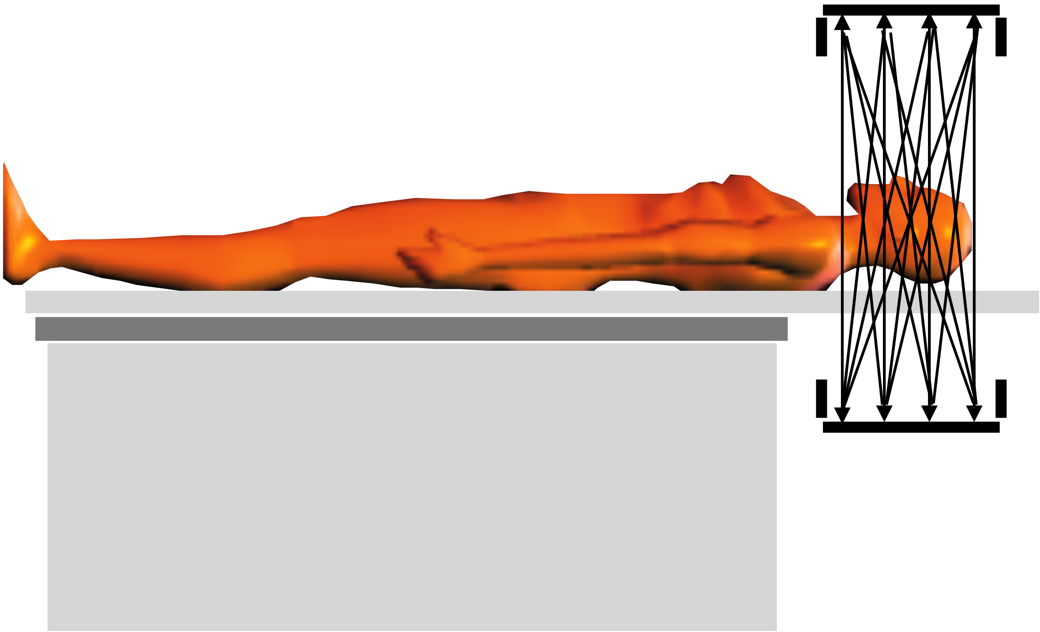
* system matrix still to be improved (motion, positron path, collimator penetration in SPECT, aso)
* hot topic: efficient and robust regularization

Iterative reconstruction in X-ray CT

Companies are now developing iterative reconstruction in X-ray CT, while it was used only in SPECT and PET so far. Why?

Beyond 2D reconstruction…





Solution : « fully 3D reconstruction »

Three types of fully 3D reconstruction

- Analytical method

  3D FBP : extension of the 2D FBP

- Methodes of rebinning

  rearrangement of the 3D data to make
  2D reconstruction algorithms applicable

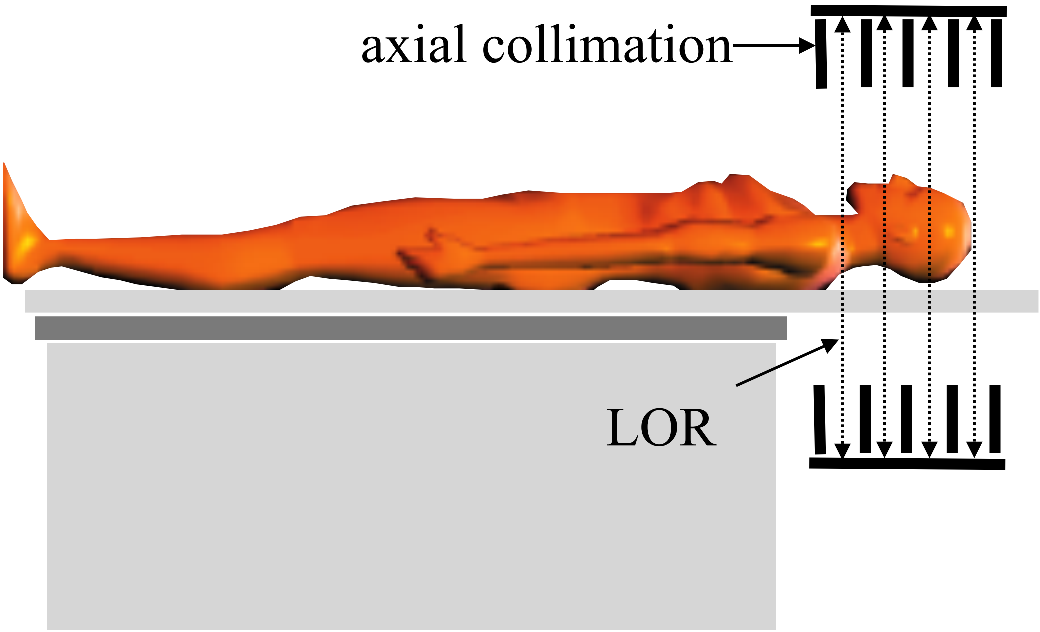
- Iterative methods

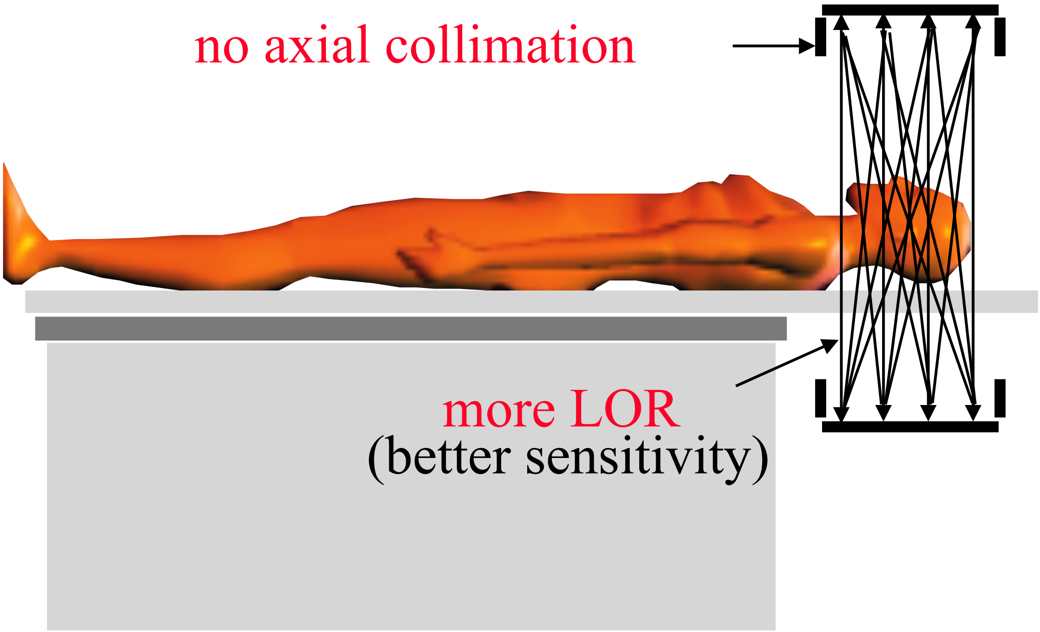
  estimation of a « fully 3D » matrix system

Analytical 3D reconstruction

- 3D FBP : extension of 2D FBP

\- accounts for data redundancy

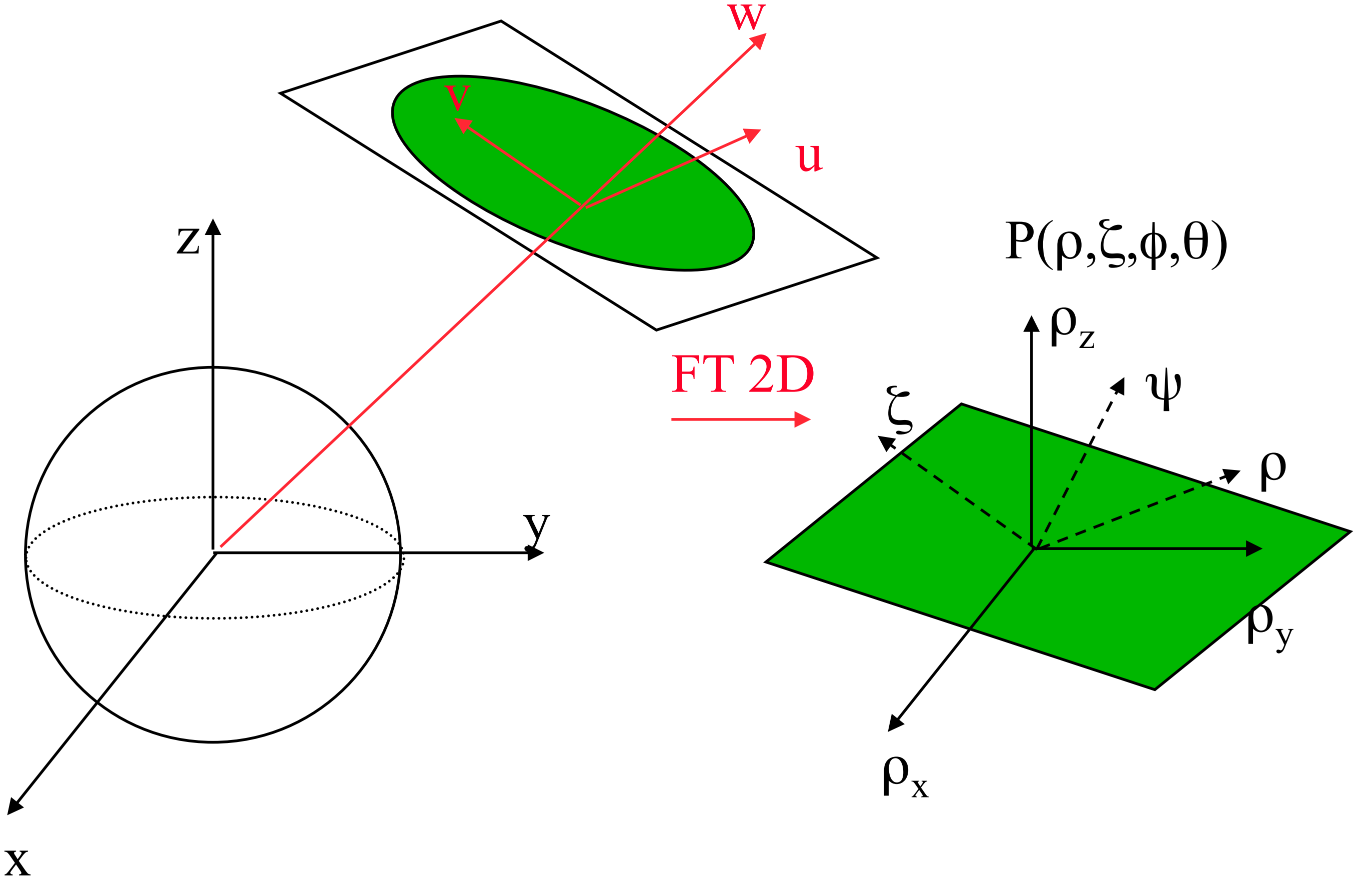




Central slice theorem in 3D

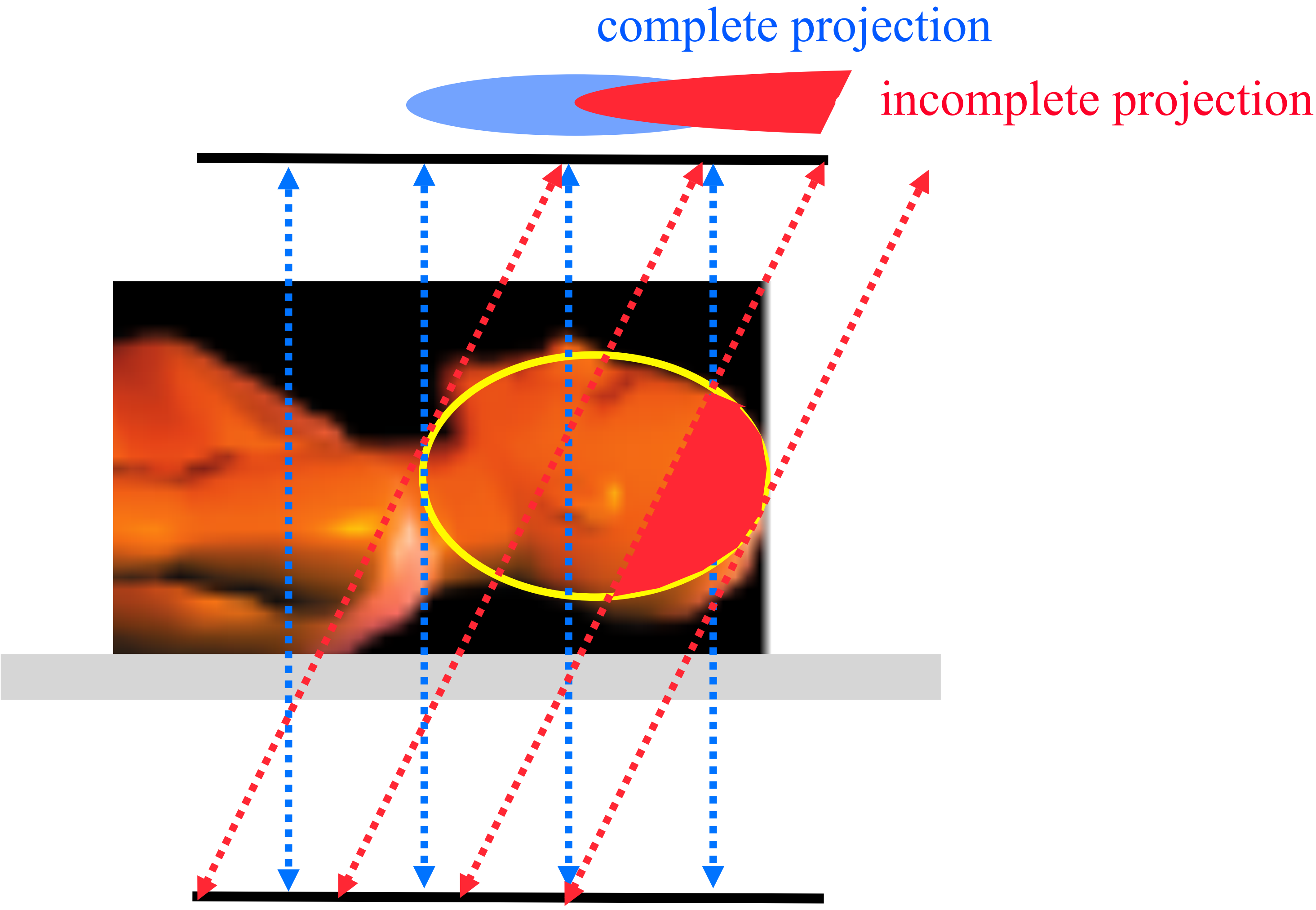
Similar to the 2D version

projection = $p(u,v,\phi,\theta)$

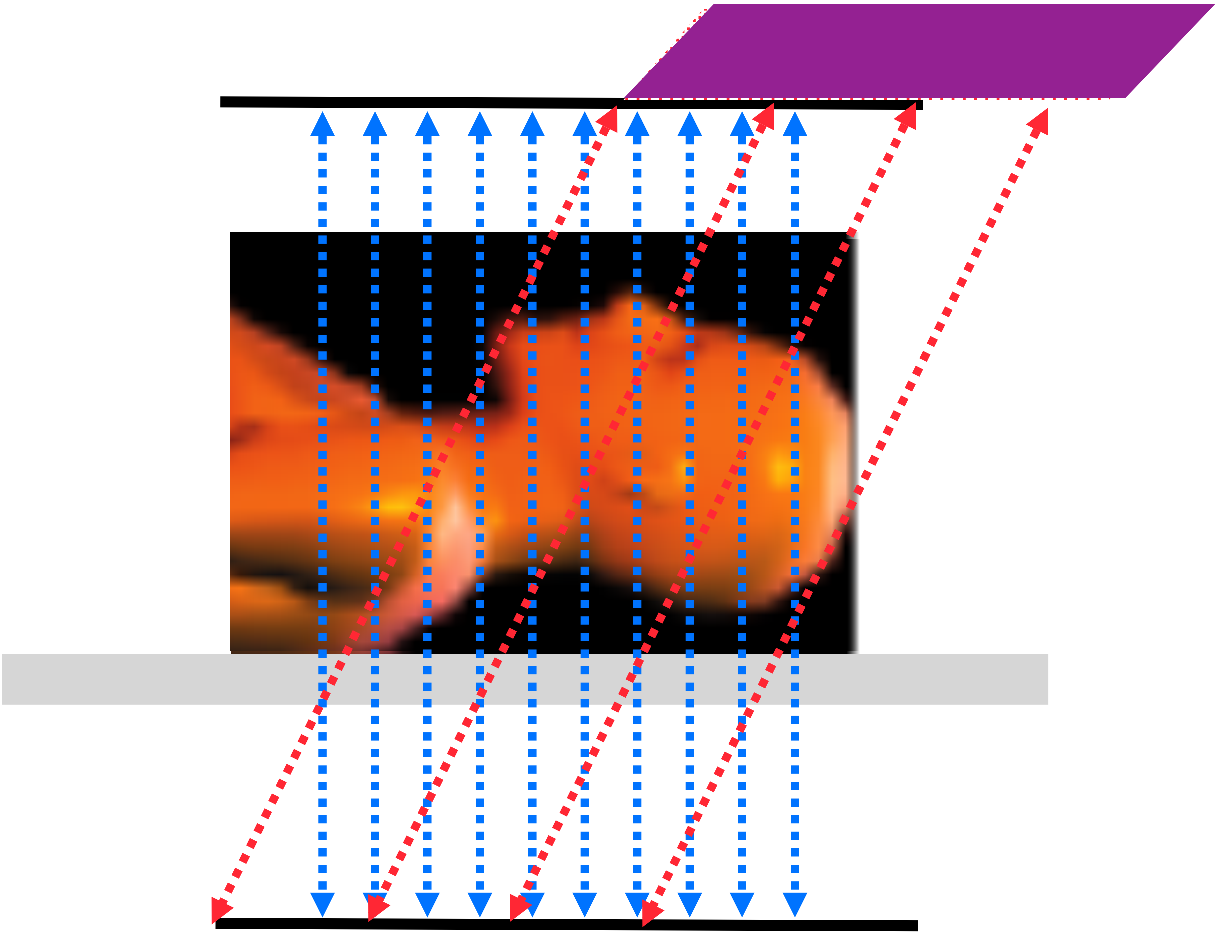


object = $f(x,y,z)$ $\xrightarrow{\text{FT 3D}}$ $P(\rho,\theta) = F(\rho_x, \rho_y, \rho_z)\Big|_{\psi=0}$

… but 3D FBP requires “complete” data



3D backprojection using incomplete data



- Extraction of the 2D data (disregarding the oblique LOR)

- Reconstruction of a first estimate of f using 2D FBP

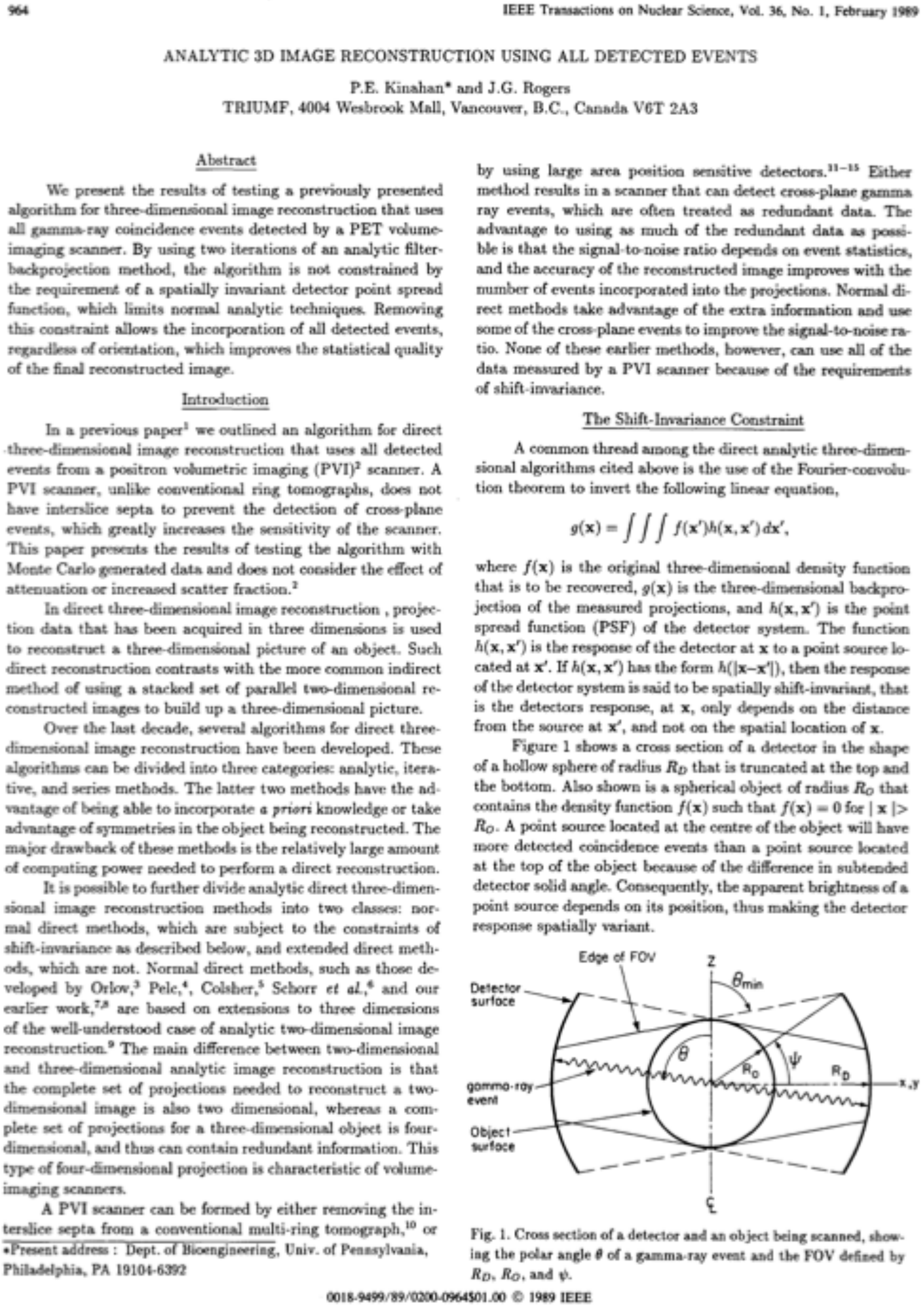
- Estimation of the truncated data by forward projection of the estimated f

- Merging the estimated f and the available measurements

- Once the data are complete, use 3D FBP

3D backprojection using incomplete data

- This is the classical method of 3D reprojection (3DRP, 3D reprojection method, Kinahan and Rogers, IEEE Trans Nucl Sci 1989)



964 IEEE Transactions on Nuclear Science, Vol. 36, No. 1, February 1989

ANALYTIC 3D IMAGE RECONSTRUCTION USING ALL DETECTED EVENTS

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Abstract

We present the results of testing a previously presented algorithm for three-dimensional image reconstruction that uses all gamma-ray coincidence events detected by a PET volume-imaging scanner. By using two iterations of an analytic filter-backprojection method, the algorithm is not constrained by the requirement of a spatially invariant detector point spread function, which limits normal analytic techniques. Removing this constraint allows the incorporation of all detected events, regardless of orientation, which improves the statistical quality of the final reconstructed image.

Introduction

In a previous paper[1] we outlined an algorithm for direct three-dimensional image reconstruction that uses all detected events from a positron volumetric imaging (PVI)[2] scanner. A PVI scanner, unlike conventional ring tomographs, does not have interslice septa to prevent the detection of cross-plane events, which greatly increases the sensitivity of the scanner. This paper presents the results of testing the algorithm with Monte Carlo generated data and does not consider the effect of attenuation or increased scatter fraction.[2]

In direct three-dimensional image reconstruction , projection data that has been acquired in three dimensions is used to reconstruct a three-dimensional picture of an object. Such direct reconstruction contrasts with the more common indirect method of using a stacked set of parallel two-dimensional reconstructed images to build up a three-dimensional picture.

Over the last decade, several algorithms for direct three-dimensional image reconstruction have been developed. These algorithms can be divided into three categories: analytic, iterative, and series methods. The latter two methods have the advantage of being able to incorporate *a priori* knowledge or take advantage of symmetries in the object being reconstructed. The major drawback of these methods is the relatively large amount of computing power needed to perform a direct reconstruction.

It is possible to further divide analytic direct three-dimensional image reconstruction methods into two classes: normal direct methods, which are subject to the constraints of shift-invariance as described below, and extended direct methods, which are not. Normal direct methods, such as those developed by Orlov,[3] Pelc,[4], Colsher,[5] Schorr *et al.*,[6] and our earlier work,[7,8] are based on extensions to three dimensions of the well-understood case of analytic two-dimensional image reconstruction.[9] The main difference between two-dimensional and three-dimensional analytic image reconstruction is that the complete set of projections needed to reconstruct a two-dimensional image is also two dimensional, whereas a complete set of projections for a three-dimensional object is four-dimensional, and thus can contain redundant information. This type of four-dimensional projection is characteristic of volume-imaging scanners.

A PVI scanner can be formed by either removing the interslice septa from a conventional multi-ring tomograph,[10] or by using large area position sensitive detectors.[11-15] Either method results in a scanner that can detect cross-plane gamma ray events, which are often treated as redundant data. The advantage to using as much of the redundant data as possible is that the signal-to-noise ratio depends on event statistics, and the accuracy of the reconstructed image improves with the number of events incorporated into the projections. Normal direct methods take advantage of the extra information and use some of the cross-plane events to improve the signal-to-noise ratio. None of these earlier methods, however, can use all of the data measured by a PVI scanner because of the requirements of shift-invariance.

The Shift-Invariance Constraint

A common thread among the direct analytic three-dimensional algorithms cited above is the use of the Fourier-convolution theorem to invert the following linear equation,

$$g(\mathbf{x}) = \int\int\int f(\mathbf{x}')h(\mathbf{x},\mathbf{x}')\,d\mathbf{x}',$$

where $f(\mathbf{x})$ is the original three-dimensional density function that is to be recovered, $g(\mathbf{x})$ is the three-dimensional backprojection of the measured projections, and $h(\mathbf{x},\mathbf{x}')$ is the point spread function (PSF) of the detector system. The function $h(\mathbf{x},\mathbf{x}')$ is the response of the detector at $\mathbf{x}$ to a point source located at $\mathbf{x}'$. If $h(\mathbf{x},\mathbf{x}')$ has the form $h(|\mathbf{x}-\mathbf{x}'|)$, then the response of the detector system is said to be spatially shift-invariant, that is the detectors response, at $\mathbf{x}$, only depends on the distance from the source at $\mathbf{x}'$, and not on the spatial location of $\mathbf{x}$.

Figure 1 shows a cross section of a detector in the shape of a hollow sphere of radius R_D that is truncated at the top and the bottom. Also shown is a spherical object of radius R_O that contains the density function $f(\mathbf{x})$ such that $f(\mathbf{x}) = 0$ for $|\,\mathbf{x}\,| > R_O$. A point source located at the centre of the object will have more detected coincidence events than a point source located at the top of the object because of the difference in subtended detector solid angle. Consequently, the apparent brightness of a point source depends on its position, thus making the detector response spatially variant.

Fig. 1. Cross section of a detector and an object being scanned, showing the polar angle θ of a gamma-ray event and the FOV defined by R_D, R_O, and ψ.

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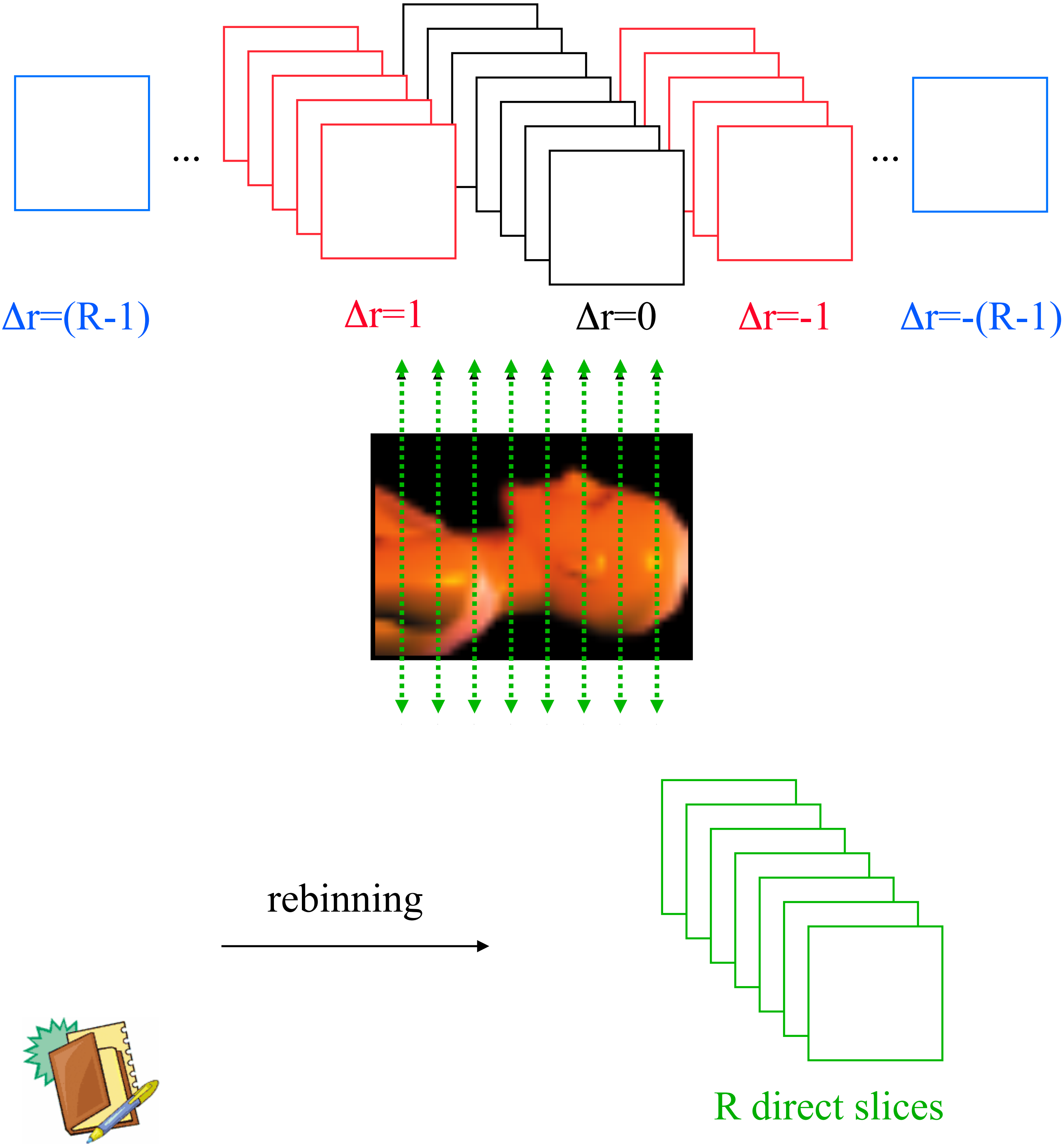
Rebinning methods

- Using R^2 sinograms (R number of detector rings), estimation of 2R-1 sinograms corresponding to direct slices



…

…

Δr=(R-1)

Δr=1

Δr=0

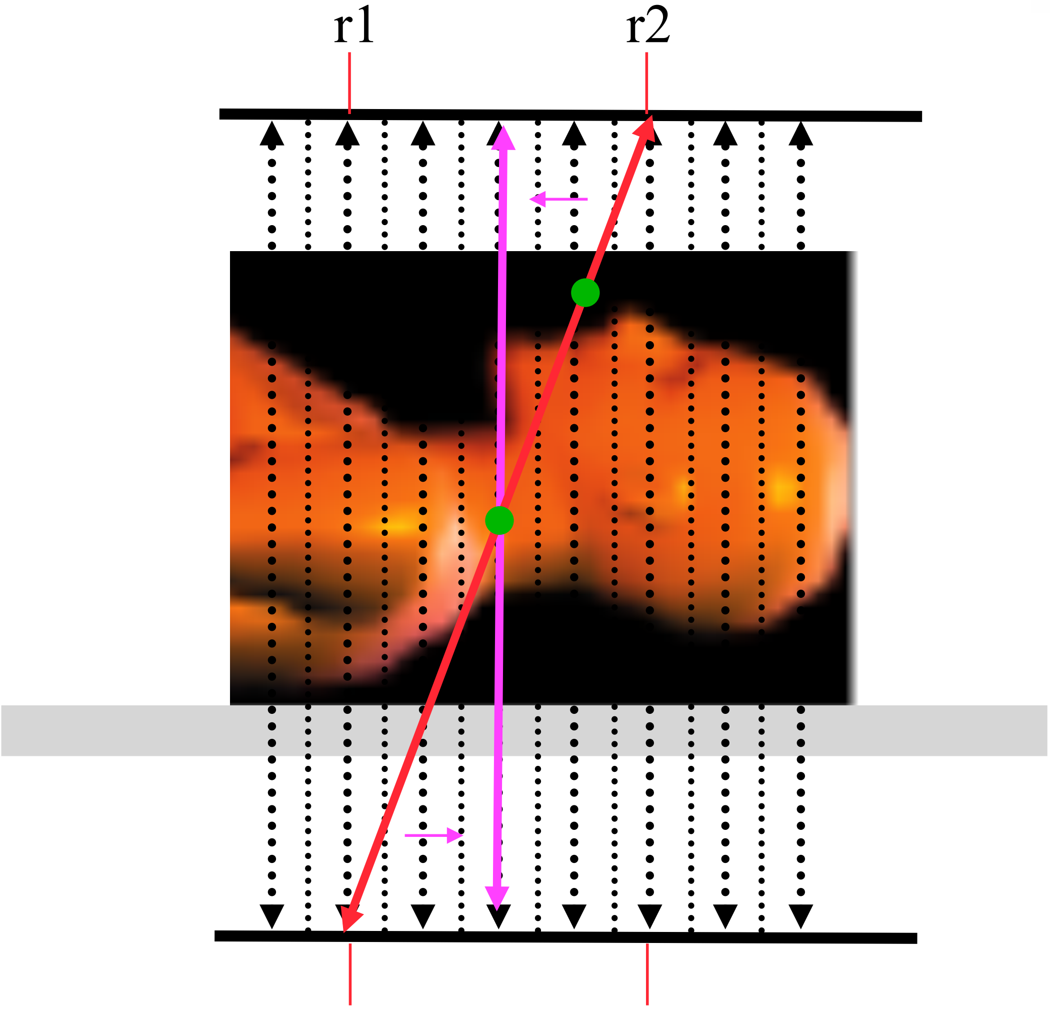
Δr=-1

Δr=-(R-1)

rebinning

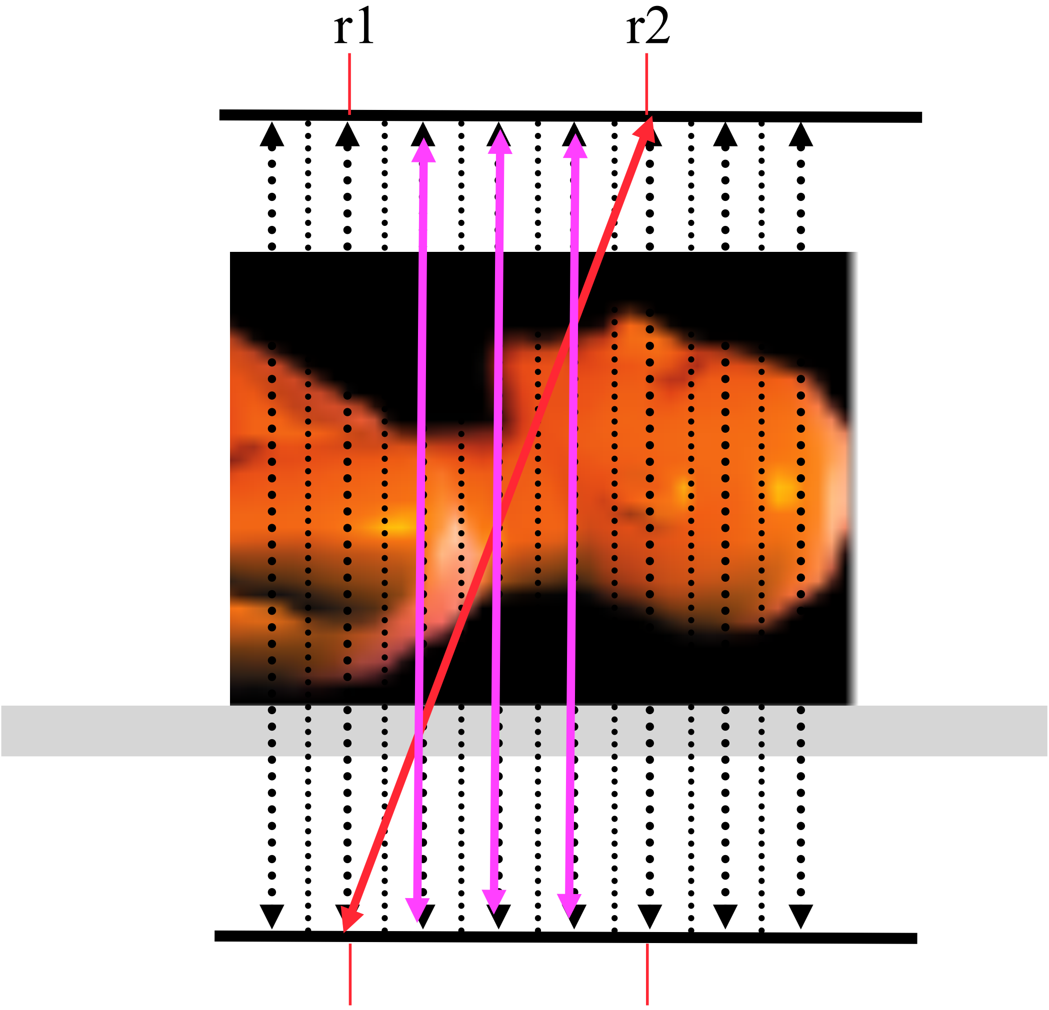
R direct slices

Single slice rebinning



r_rebin = (r1+r2)/2

Multi slice rebinning



FORE

FOurier REbinning (1995)

Oblique sinograms are resampled in the 2D Fourier domain to reassign events to direct slices

After rebinning

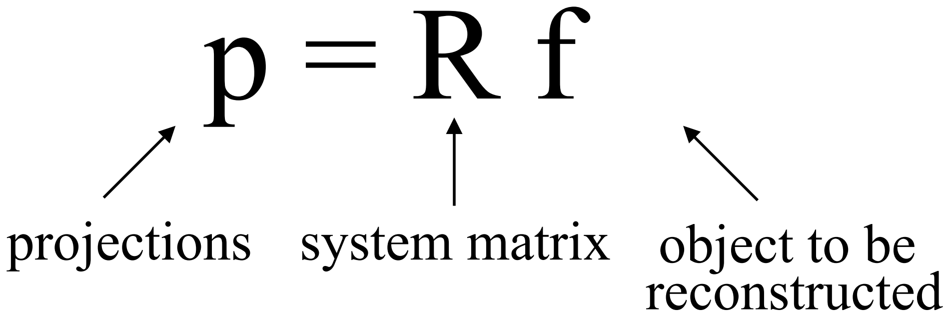
Use of a 2D reconstruction method (either FBP or iterative)

Rebinning

- Also called 2.5 D reconstruction. Why ?

Iterative discrete methods

- Identical to 2D reconstruction



$$p = R\,f$$

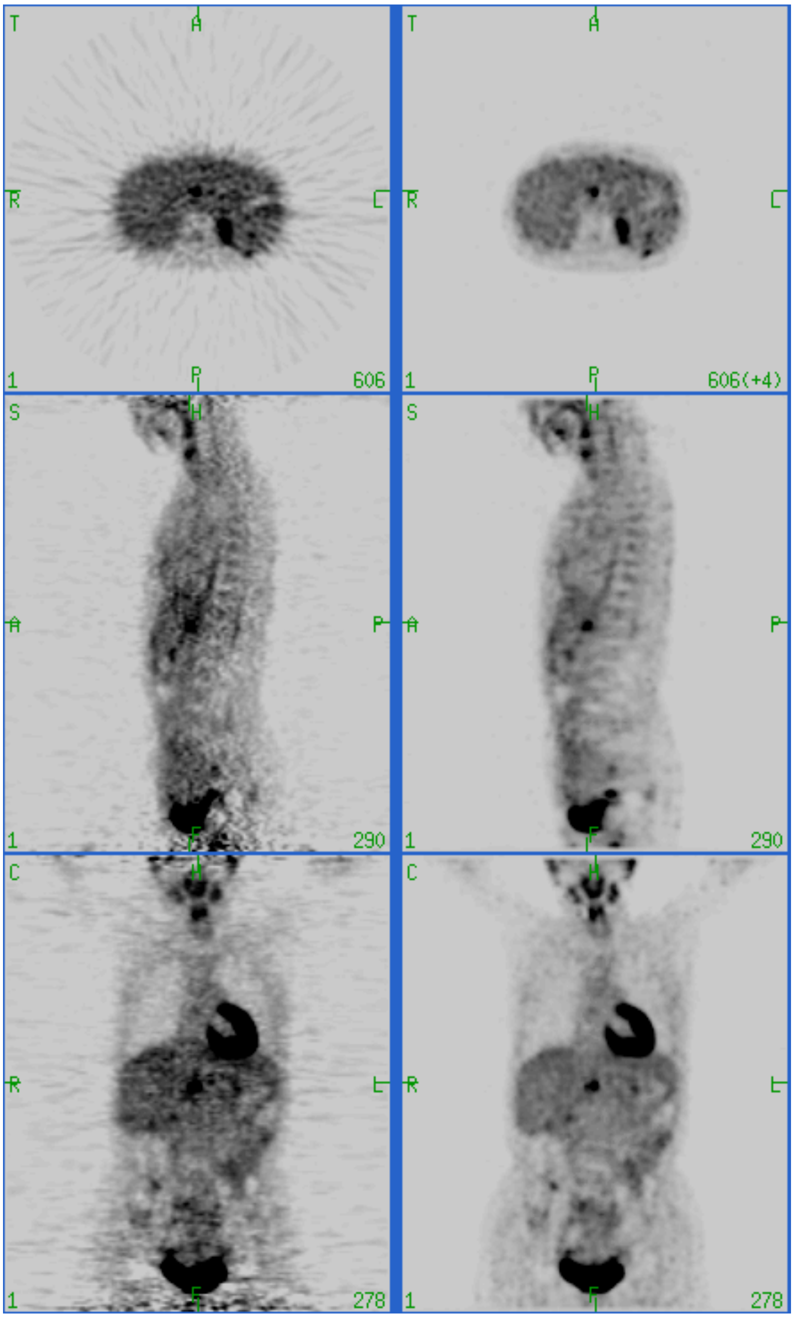
projections system matrix object to be reconstructed

- Challenges :

- size of R (> 10 million LOR in 3D PET)

- accurate estimate of R in 3D to account to all physics and geometry effects (scatter, detector response function, axially variable sensitivity, aso)

Why is « fully 3D » reconstruction appealing?

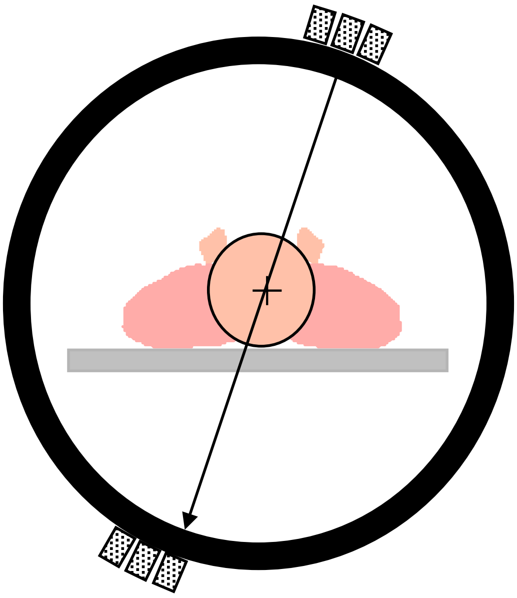


FORE-FBP 3D Ramla

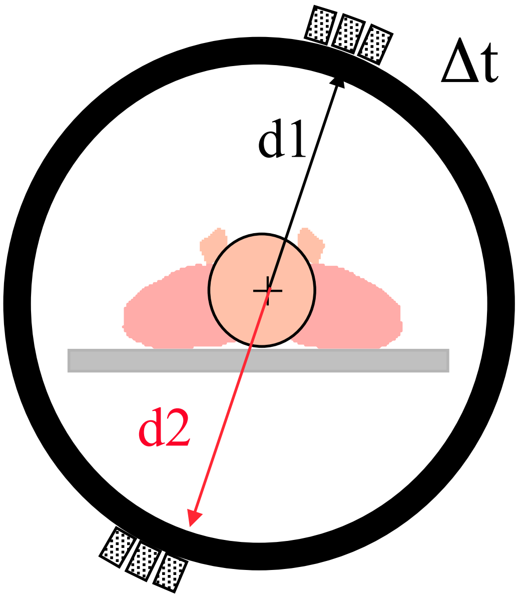
Joel Karp, University of Pennsylvanie

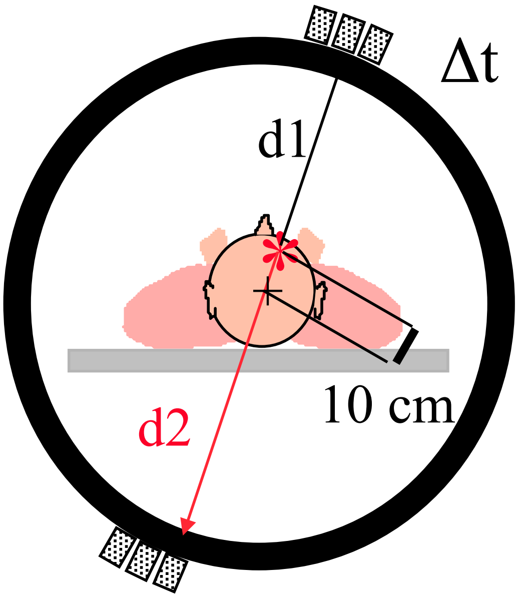
PET reconstruction using time of flight

What does that change?



Without TOF, no information regarding the annihilation point on the LOR





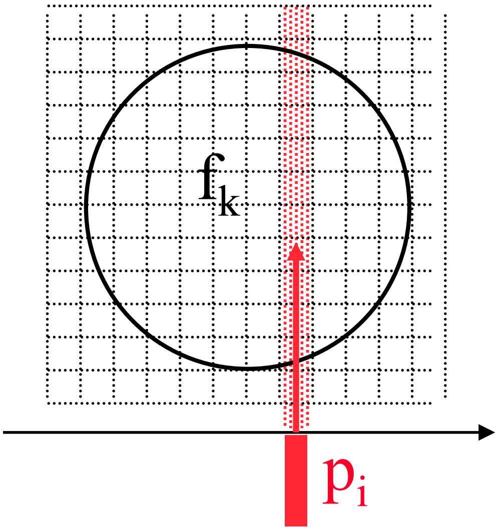
If Δt = 0, d1 = d2, annihilation took place at the exact center of the LOR

If Δt = 667 ps, d1-d2=20cm, annihilation took place 10 cm off-centered

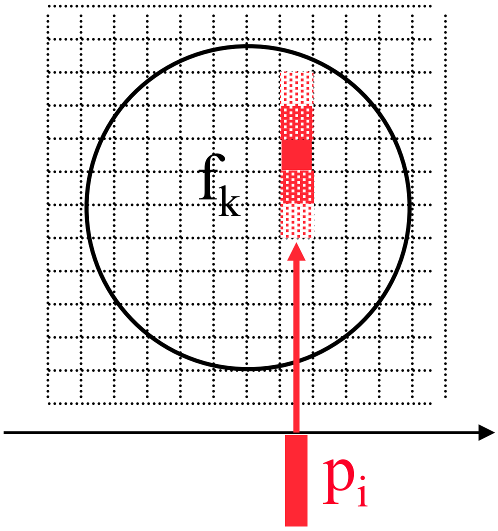
The precision with which Δt, hence d1-d2, can be measured is limited

PET reconstruction using time of flight

A priori regarding the location of the annihilation on the LOR can be modelled during backprojection



Backprojection without prior

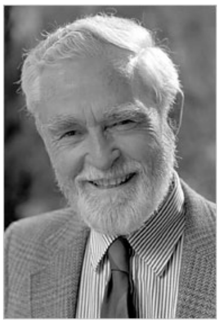


Backprojection with prior

The reconstructed signal will thus be concentrated on smaller regions from which it is likely it has been emitted: increase in contrast to noise ratio (less background activity)

Historical data

- 1917 : Johann Radon : "*About the determination of functions from their integral functions in certain directions*" Mathematic work, no application



1921-2007

- 1956 : Bracewell : demonstration of the relationships between Fourier transform and Radon transform
- 1963 : first applications to medical tomography
  - Kuhl, prof of radiology : first images obtained by backprojection
  - Cormack, physicist : application of Radon results to X-ray acquisitions

1929-

- 1970 : publication of the first CT image
- 1970-73 : design of the first CT scanner by Cormack and Hounsfield
- 1979 : Cormack et Hounsfield won the Nobel prize in Medicine

1924-1998

1919-2004

Which method for which application?

- X-ray CT

 * FBP thanks to excellent signal to noise ratio in the data
 * but iterative reconstruction is entering this application field to decrease the dose required to get excellent images : « low dose » CT

- SPECT

* FBP only for a long time (before OSEM was invented)
* Now : iterative algorithms, in particular OSEM, to:
  - reduce streak artefacts,
  - improve quantitative accuracy
  - better deal with low stat (10 000 less events than in CT)
  - current computers and GPU make iterative reconstruction usable in the clinics

- PET

* FBP, MLEM, OSEM, RAMLA

What you should now know

- Key aspects in tomography:

- differences between emission tomography and transmission tomography

- the fact that the mathematical problem is actually exactly the same in emission and trasmission tomography

- the function that is reconstructed in CT, PET and SPECT

- why tomography gives more information than planar imaging

- why tomograhic reconstruction is an ill-posed problem

- that there are two families of reconstruction methods, and how they differ

- the difference between reconstruction and “fully 3D reconstruction”

What you should now know

- Reconstruction techniques:

- differences between projections and sinograms

- principle of filteted backprojection

- the role of the filter in FBP

- why the Ramp filter is not sufficient

- how the parameter filter can affect the resulting images

- the principle of iterative reconstruction

- the parameters that can be changed in iterative reconstruction to improve image quality / accuracy

- what is a system matrix

- the principle of OSEM / MLEM

- the properties of MLEM and OSEM

- the 3 types de fully 3D reconstruction

- what is the rebinning in TEP

To know more ...

Short articles

- Analytic and iterative reconstruction algorithms in SPECT. Journal of Nuclear Medicine 2002, 43:1343-1358

- J. Qi and R. Leahy, Iterative reconstruction techniques in emission tomography,
Topical review, Phys. Med. Biol. , vol. 51 (2006), R541-R578

- Articles you can download on:
http://www.guillemet.org/irene/equipe4/cours.html

Books

- Kinahan PE, Defrise M, and Clackdoyle R. Analytic image reconstruction methods. In: Emission Academic Press, 2004
- Zeng GL, Medical Image reconstruction: a conceptual tutorial, Springer, 2009

Your questions