



# A short reader's guide to 3D tomographic reconstruction

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## Abstract

This text summarizes the main technical problems related to 3D image reconstruction in PET, SPECT and CT, and provides references to a selection of key papers and to review papers. © 2001 Elsevier Science Ltd. All rights reserved.

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Traditionally, tomography has considered the 3D object under study as a stack of parallel slices, and each slice was measured and reconstructed independent of the other slices (see Ref. [1] for the history of tomography). In SPECT, PET and CT the data acquired for each slice are well modeled as line integrals of the distribution function to be reconstructed (this function is the local concentration of tracers in SPECT and PET, or the linear X-ray attenuation coefficient in CT). The set of measured line integrals corresponds to the 2D Radon transform of a slice, and the image reconstruction problem then consists of recovering a 2D function from its Radon transform. Classical books on 2D tomography are still relevant [2–5] except for the chapters on iterative techniques and on 3D tomography.

This standard, 2D approach to tomography makes poor utilization of the radiation available since only photons (gamma rays or X-rays) that are flying within the plane of a slice can be used for reconstruction. Collimation of the detector and also of the X-ray source in CT must then be used to stop the other, “oblique” photons. The rationale for three-dimensional tomography is to improve the sensitivity of the scanner by relaxing or by completely removing collimation in such a way that photons traversing several slices are also detected. By increasing the number of photons detected, this 3D approach allows faster imaging, but image reconstruction becomes much more complex since the 3D object can no longer be separated into a stack of independent slices as with the traditional 2D approach to tomography. Another disadvantage of all 3D tomographic techniques is an increased background due to photons that have undergone scattering.

The idea of 3D tomography is not new [1]. For instance, some early PET scanners in the 1960s were actually 3D scanners, and a cone-beam CT scanner was already developed in the early 1980s at the Mayo clinic. Note also that the very old longitudinal tomography (tomosynthesis) is inherently 3D. However, it is only after about 1990 that developments in detector technology (block detectors for PET,<sup>1</sup> flat panel area X-ray detectors [6], etc.), in 3D algorithms and in the available computer power have allowed practical applications of 3D tomography. Many recent algorithmic developments are described in the Proceedings of the five International Symposia on Fully Three Dimensional Reconstruction in Radiology and Nuclear Medicine [7–11].

## 1. Positron emission tomography

In PET, 3D tomography has become the standard approach. Most modern scanners have a cylindrical geometry, typically with a radius of 40 cm and an axial field-of-view of 15 to 20 cm. They can be operated in the so-called 3D mode, without any collimation (except external shields for the activity located outside the FOV). Compared to the 2D acquisition mode where the same scanner is operated with annular septa, the sensitivity is multiplied by factors of the order of five. The performance of various 3D scanners is described in Refs. [12–14]. Refs. [15,16] present a different type of scanner based on a rotating pair of gamma cameras; these cameras are used without collimators and here again the data collected are fully 3D. The general principles of 3D acquisition in PET are discussed in detail in Refs. [17,18]

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<sup>1</sup> Most key references on block detectors have appeared in the IEEE Transactions in Nuclear Sciences.

(see Ref. [19] for a review), and the various data correction procedures (especially for the scatter, which is about three to five times larger than in 2D mode) in Ref. [20].

From the point of view of image reconstruction, a 3D PET scanner measures data that can be considered (after appropriate corrections) as 2D parallel projections of the tracer distribution along a set of directions in space that depend on the geometry of the scanner. The problem of 3D image reconstruction from parallel projections is well understood [21–23], and has applications in other fields such as X-ray micro-tomography and electron microscopy for the study of the structure of macromolecules [24,25]. The standard algorithm, used by most 3D PET scanners, is based on filtered-backprojection [26]. Recent research has concentrated in two directions:

- The design of fast algorithms to cope with the rapidly increasing size of the collected data, especially for whole-body studies. These algorithms are called rebinning algorithms because they reduce the 3D data set to a stack of independent 2D data sets for each transaxial slice. By factoring the problem, rebinning algorithms accelerate 3D reconstruction, and also reduce the data size, by an order of magnitude [27–29].
- The development of iterative reconstruction techniques that better model the statistical properties of the measurement noise, specifically Poisson noise. These algorithms produce a sequence of image estimates that approach the image that maximizes some cost function. The cost function is usually the likelihood function, that is the probability that the image would produce the specific data that have been measured. The cost function may also involve penalty terms that favor images that satisfy smoothness constraints; these penalty terms avoid excessive noise amplification [30,31]. The literature on iterative algorithms for PET is extremely wide, and unfortunately no recent review paper is available. Slides of an excellent introductory lecture by J Fessler can be found in Ref. [32]. The most popular algorithm is by far the Ordered Subset Expectation Maximization (OSEM) method [33], but this is partly due to the fact that it is extremely easy to implement; the superiority of this algorithm over other iterative methods has not been clearly demonstrated. A 3D version of this method is described in Ref. [34].

## 2. Single photon emission tomography

Most SPECT scanners are equipped with parallel hole or fan-beam collimators. Both types of collimators separate the 3D object into a stack of 2D slices. Completely suppressing collimation is of course impossible in SPECT since collimation is required to define the line of flight of the detected gamma rays. However, the sensitivity can be significantly improved by using converging collimators such as cone-

beam collimators (discussed in Ref. [2], see also Ref. [35] for a review), pinhole collimators [2,36–38], or more complicated structures such as the cardio-focal collimator [39]. Note that the sensitivity improvement thus achieved with respect to parallel and fan-beam collimators, for a given spatial resolution, is smaller than in PET and that the price to pay is always a reduction of the size of the field-of-view. Thus, as in 2D SPECT, collimator design requires a compromise between sensitivity, spatial resolution and size of the field-of-view. Because of the smaller field-of-view, converging collimators are used to image small organs such as the heart. Pinhole SPECT proves useful for the thyroid (see e.g. Ref. [40]) and for small animal studies (see e.g. Ref. [41]).

With all converging collimators (except fan-beam collimators), the data collected cannot be separated into independent 2D slices, and fully 3D reconstruction algorithms are required. In fact, even with parallel and fan-beam collimators, an accurate modeling of the collimator response and of the scatter background prevents any factorization of the problem in independent slices. In this sense, one should always regard SPECT as a fully 3D problem. Because of the lower sensitivity of SPECT scanners compared to PET, and also because the data are not well modeled as line integrals of the tracer distribution (the collimator holes define conical tubes of response rather than lines), the best results are obtained using iterative reconstruction algorithms. Indeed these algorithms are able to accurately model the variation of the spatial resolution with the distance from the collimator, the scatter background and attenuation, and the statistical properties of the low-count data (see Ref. [42] for a state-of-the-art review). Note however that many algorithms used in practice and proposed by manufacturers only take into account part of these physical effects. Another remark is that in principle the iterative algorithms for 3D SPECT do not differ from those developed for PET [30–33].

## 3. Cone-beam X-ray tomography

CT scanners equipped with multiple rows of detectors have been recently introduced by Siemens and GE (information can be found on their web-site). By combining the rotation of the gantry with a uniform translation of the couch, the X-ray source moves on a helical path with respect to the patient, and for each position of the X-ray source along this path, data are collected on the multi-row detector, allowing much faster imaging than the standard spiral CT scanners, which also use a helical motion of the source with respect to the patient, but have only one row of detectors [43,44]. Geometrically a multi-row scanner measures cone-beam projections of the patient, but these projections are severely truncated in the axial direction owing to the small number of rows (four in the current scanners). Future CT scanners will feature a larger number of rows, thereby

allowing still faster imaging. Cone-beam scanners based on large area detectors have already been used for high contrast imaging [45] and are fairly common for industrial non-destructive testing applications.

Image reconstruction for cone-beam CT scanners is extremely complex and is the object of intensive research. Good approximate algorithms have been proposed that are appropriate for the current scanners [46,47] because the axial angular aperture of the cone-beam of X-rays is very small, but exact algorithms will be needed in the future. Recent progresses in this direction are discussed in Refs. [48,49] and recent overviews of cone-beam reconstruction can be found in Refs. [50,51].

#### 4. Summary

Organs have long been considered as a stack of parallel slices in tomographical medical imaging, where each slice was measured and reconstructed independent of the other slices. This standard, 2D approach to tomography makes poor utilization of the radiation available since only photons (gamma rays or X-rays) that are flying within the plane of a slice can be used for reconstruction. Collimation of the detector and also of the X-ray source in CT must then be used to stop the other, “oblique” photons. Three-dimensional tomography improves the sensitivity of the scanner by relaxing or by completely removing collimation in such a way that photons traversing several slices are also detected. By increasing the number of photons detected, this 3D approach allows faster imaging, but image reconstruction becomes much more complex since the 3D object can no longer be separated into a stack of independent slices. This paper lists the main research topics in 3D reconstruction for three application fields: positron emission tomography, where the 3D acquisition is now standard, single-photon emission tomography with pinhole or cone-beam collimators, and radiology with the new generation of multi-row CT scanners.

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